

MFM2P – Linear Systems – Day 6: Solving by Substitution Part II

Solve each system of linear equations

1. $x + y + 4 = 0$ and $4x - 2y - 2 = 0$

① $x + y + 4 = 0$
 $\cancel{+x} \quad \cancel{+y} \quad \cancel{+4} \quad -x - 4$
 $\boxed{y = -x - 4}$

$4x - 2y - 2 = 0$
 $\cancel{-4x} \quad \cancel{+2y} \quad \cancel{-2} \quad -4x + 2$
 $\frac{-2y}{-2} = \frac{-4x + 2}{-2}$

$\boxed{y = 2x - 1}$

② $-x - 4 = 2x - 1$
 $\cancel{-2x} \quad \cancel{+2x}$
 $-3x - 4 = -1$
 $\cancel{+4} \quad \cancel{+4}$
 $\frac{-3x}{-3} = \frac{3}{-3}$
 $\underline{x = -1}$

③ $y = -(-1) - 4$
 $y = 1 - 4$
 $y = -3$

$y = 2(-1) - 1$
 $y = -2 - 1$
 $y = -3$

\therefore the POI is $(-1, -3)$

2. $C = -3d + 7$ and $2d - C = -2$

① $\boxed{C = -3d + 7}$

$2d - C = -2$
 $\cancel{-2d} \quad \cancel{-2d}$

$\frac{-C}{-1} = \frac{-2d - 2}{-1}$

$\boxed{C = 2d + 2}$

② $-3d + 7 = 2d + 2$
 $\cancel{-2d} \quad \cancel{-2d}$
 $-5d + 7 = 2$
 $\cancel{+7} \quad \cancel{+7}$
 $\frac{-5d}{-5} = \frac{-5}{-5}$
 $\underline{d = 1}$

③ $C = -3(1) + 7$
 $C = -3 + 7$
 $C = 4$

$C = 2(1) + 2$
 $C = 2 + 2$
 $C = 4$

\therefore the POI is $(1, 4)$

① 3. $-2x + 4y = 0$ and $x + y = -6$.

$$\begin{array}{r} -2x + 4y = 0 \\ +2x \quad +2x \end{array}$$

$$\frac{4y}{4} = \frac{2x}{4}$$

$$y = 0.5x$$

$$\begin{array}{r} x + y = -6 \\ -x \quad -x \end{array}$$

$$y = -x - 6$$

② $0.5x = -x - 6$

$$\begin{array}{r} +x \quad +x \end{array}$$

$$\frac{1.5x}{1.5} = \frac{-6}{1.5}$$

$$x = -4$$

③ $y = 0.5(-4)$ $y = -(-4) - 6$
 $y = -2$ $y = 4 - 6$
 $y = -2$

∴ the POI is $(-4, -2)$

① 4. $2x - 2y = 8$ and $5x + y = 2$.

$$\begin{array}{r} 2x - 2y = 8 \\ -2x \quad -2x \end{array}$$

$$\frac{-2y}{-2} = \frac{-2x + 8}{-2}$$

$$y = x - 4$$

$$\begin{array}{r} 5x + y = 2 \\ -5x \quad -5x \end{array}$$

$$y = -5x + 2$$

② $x - 4 = -5x + 2$

$$\begin{array}{r} +5x \quad +5x \end{array}$$

$$\frac{6x - 4}{6} = \frac{2}{6}$$

$$\frac{6x}{6} = \frac{6}{6}$$

$$x = 1$$

$$\begin{array}{l} x = 1(1) - 4 \\ y = 1 - 4 \\ y = -3 \end{array}$$

$$\begin{array}{l} y = -5(1) + 2 \\ y = -5 + 2 \\ y = -3 \end{array}$$

∴ POI is $(1, -3)$

5. $6x - 3y = -21$ and $2x + 8y = -16$.

$$\begin{array}{r} 6x - 3y = -21 \\ -6x \quad -6x \\ \hline -3y = -6x - 21 \\ \frac{-3y}{-3} = \frac{-6x}{-3} - \frac{21}{3} \end{array}$$

$$\boxed{y = 2x + 7}$$

$$\begin{array}{r} 2x + 8y = -16 \\ -2x \quad -2x \\ \hline \end{array}$$

$$\frac{8y}{8} = \frac{-2x}{8} - \frac{16}{8}$$

$$\boxed{y = -0.25x - 2}$$

$$\begin{array}{r} 2x + 7 = -0.25x - 2 \\ +0.25x \quad +0.25x \end{array}$$

$$\begin{array}{r} 2.25x + 7 = -2 \\ -7 \quad -7 \\ \hline \end{array}$$

$$\begin{array}{r} 2.25x = -9 \\ \frac{2.25x}{2.25} = \frac{-9}{2.25} \end{array}$$

$$\underline{x = -4}$$

$$\begin{array}{l} (3) \quad y = 2(-4) + 7 \\ y = -8 + 7 \\ y = -1 \end{array}$$

$$\begin{array}{l} y = -0.25(-4) - 2 \\ y = 1 - 2 \\ y = -1 \end{array}$$

$$\therefore \text{POI is } \underline{\underline{(-4, -1)}}$$

6. $2x - 4y = -12$ and $4x + y = -6$.

$$\begin{array}{r} 2x - 4y = -12 \\ -2x \quad -2x \\ \hline \end{array}$$

$$\begin{array}{r} -4y = -2x - 12 \\ \frac{-4y}{-4} = \frac{-2x}{-4} - \frac{12}{4} \end{array}$$

$$\boxed{y = 0.5x + 3}$$

$$\begin{array}{r} 4x + y = -6 \\ -4x \quad -4x \\ \hline \end{array}$$

$$\boxed{y = -4x - 6}$$

$$\begin{array}{l} (3) \quad y = 0.5(-2) + 3 \\ y = -1 + 3 \\ y = 2 \end{array}$$

$$\begin{array}{l} y = -4(-2) - 6 \\ y = 8 - 6 \\ y = 2 \end{array}$$

$$\therefore \text{POI is } \underline{\underline{(-2, 2)}}$$

$$\begin{array}{r} 0.5x + 3 = -4x - 6 \\ +4x \quad +4x \end{array}$$

$$\begin{array}{r} 4.5x + 3 = -6 \\ -3 \quad -3 \\ \hline \end{array}$$

$$\begin{array}{r} 4.5x = -9 \\ \frac{4.5x}{4.5} = \frac{-9}{4.5} \end{array}$$

$$\underline{x = -2}$$

7. Lydia works at an electronics store. Her annual salary is represented by the equation $S = 26\,500 + 20n$ where n is the number of television sets sold. Calvin works at another electronics store that pays an annual salary of $S = 28\,000 + 15n$.

- How many television sets must be sold for Lydia and Calvin to earn the same salary?
- When does Lydia earn more money than Calvin?
- When does Calvin earn more money than Lydia?

$$a.) \quad \begin{array}{r} 26\,500 + 20n \\ -15n \\ \hline \end{array} = \begin{array}{r} 28\,000 + 15n \\ -15n \\ \hline \end{array}$$

$$\begin{array}{r} 5n + 26\,500 \\ -26\,500 \\ \hline \end{array} = \begin{array}{r} 28\,000 \\ -26\,500 \\ \hline \end{array}$$

$$\frac{5n}{5} = \frac{1\,500}{5}$$

$$n = 300$$

\therefore 300 TVs must be sold for both to earn the same salary

- Lydia earns more when they sell more than 300 TVs
- Calvin earns more when they sell less than 300 TVs

8. Daisy wants high-speed internet. Company A charges an \$80 installation fee plus \$30 per month. Company B charges a \$100 installation fee plus \$25 per month.

- After how many months are the costs the same?
- Under what circumstances would Daisy choose each company?

$$a.) \quad \overset{A}{C} = 30m + 80 \qquad \overset{B}{C} = 25m + 100$$

$$\begin{array}{r} 30m + 80 \\ -25m \\ \hline \end{array} = \begin{array}{r} 25m + 100 \\ -25m \\ \hline \end{array}$$

$$\begin{array}{r} 5m + 80 \\ -80 \\ \hline \end{array} = \begin{array}{r} 100 \\ -80 \\ \hline \end{array}$$

$$\frac{5m}{5} = \frac{20}{5}$$

$$m = 4$$

\therefore after 4 months both cost the same.

- Company A is better for less than 4 months, Company B is better for more than 4 months.