Using Transformations to Graph Functions of the Form $\boldsymbol{y = af[k(x-d)] + c}$

First, it's handy to know what each of our parent functions looks like in this form:

Quadratic: $f(x) = x^2$ $\boldsymbol{g(x) = a[k(x-d)]^2 + c}$	Square Root: $f(x) = \sqrt{x}$ $\boldsymbol{g(x) = a\sqrt{k(x-d)} + c}$
Reciprocal: $f(x) = \frac{1}{x}$ $\boldsymbol{g(x) = \frac{a}{k(x-d)} + c}$	Absolute Value: $f(x) = \lvert x \rvert$ $\boldsymbol{g(x) = a\lvert k(x-d)\rvert + c}$

We also need to understand what each of our ***parameters*** (***a***, ***k***, ***d*** and ***c***) do.

You should already have a pretty good grasp on ***a***, ***d*** and ***c*** from Grade 10. Although ***d*** and ***c*** were represented by different letters, the roles that they play have not changed!
And if you understand what ***a*** does, figuring out ***k*** should be <u>no problem at all</u>!

The effects of the parameters ***a***, ***k***, ***d*** and ***c***	
a: <u>reflection in the x-axis</u> - when a is negative <u>vertical stretch or compression</u> - stretch when |a| > 1 - compression when |a| < 1	**k**: <u>reflection in the y-axis</u>* - when k is negative <u>horizontal stretch or compression</u> - compression when |k| > 1 - stretch when |k| < 1 *If already symmetrical about y-axis, reflection does nothing!
c: <u>vertical translation</u> - up when c is positive - down when c is negative	**d**: <u>horizontal translation</u> - to the right when d is positive - to the left when d is negative

As you likely already understand, if we have several parameters applied to our function at once it <u>**may**</u> experience changes in shape, orientation and location!

Applying Transformations to a Graph (see Example 1 on Page 61-63)

The order that you apply transformations to a parent function is important.

Always apply *a* and *k* before *c* and *d*!

From a table of values or a graph (it will be helpful to create a table of values)

1. Apply any **horizontal stretches or compressions**.
   - Divide the x-coordinates of your original points by the value of ***k***
2. Apply any **vertical stretches or compressions**.
   - Multiply the y-coordinates of your "new" points by the value of ***a***
3. Apply any **reflections in the x-axis or y-axis**
   - Flip graph over the y-axis if ***k*** is negative. (change the sign on your "new" x-coordinates)
   - Flip graph over the x-axis if ***a*** is negative. (change the sign on your "new" y-coordinates)
4. Apply any **horizontal shifts**.
   - Shift the graph to the right if ***d*** is positive or to the left if ***d*** is negative.
     - Add ***d*** to each x-coordinate if ***d*** is positive
     - Subtract ***d*** from each x-coordinate if ***d*** is negative
5. Apply any **vertical shifts**.
   - Shift the graph up if ***c*** is positive or down if ***c*** is negative.
     - Add ***c*** to each y-coordinate if ***c*** is positive
     - Subtract ***c*** from each y-coordinate if ***c*** is negative

Applying Transformations to an Equation (see Example 2 on Page 64-65)

The key to this is simply identifying the value of each parameter.
If you are told to apply:

- A horizontal stretch by a factor of 3; ***k*** = $\frac{1}{3}$.
- A horizontal compression by a factor of $\frac{1}{3}$; ***k*** = 3.

This is why we **divide** our x-coordinates by the value of ***k***!

- A vertical stretch by a factor of 5; ***a*** = 5.
- A vertical compression by a factor of $\frac{1}{5}$; ***a*** = $\frac{1}{5}$.
- A reflection in the y-axis; ***k*** is negative.
- A reflection in the x-axis; ***a*** is negative.
- A translation 2 units up and 3 units left; ***c*** = 2 and ***d*** = -3.
- A translation 1 unit down and 4 units right; ***c*** = -1 and ***d*** = 4