Given the demand and cost functions (in thousands) below determine

a. The Profit function

b. The break even points

c. The number of units that need to be sold to achieve the max profit

d. The max profit

e. The inverse of the profit function

f. The number of units sold to reach a profit of $31,000

$$p(x) = -2x + 25 \qquad C(x) = 3x + 17$$

Remember $R(x) = p(x)[x]$

demand (really the price per unit)

a. $P(x) = R(x) - C(x)$

profit revenue cost

$P(x) = (-2x+25)x - (3x+17)$

$P(x) = -2x^2 + 25x - 3x - 17$

$\boxed{P(x) = -2x^2 + 22x - 17}$

b. find the zeros!

$-2x^2 + 22x - 17 = 0$

(Quadratic formula!)

$x = \dfrac{+22 \pm \sqrt{22^2 - 4(-2)(-17)}}{2(-2)}$

$x = \dfrac{-22 \pm \sqrt{484 + 136}}{-4}$

$x = \dfrac{-22 \pm \sqrt{348}}{-4}$

$x = \dfrac{-22 \pm 18.7}{-4}$

$x = \dfrac{-22 + 18.7}{-4}$ and $\dfrac{-22 - 18.7}{-4}$

$x : 0.825$ and 10.175

825 units

10,175 units

c) $\frac{825 + 10{,}175}{2}$

$= 5{,}500$ units must be sold to reach max profit.

d) $P(5.5) = -2(5.5)^2 + 22(5.5) - 17$

$= -60.5 + 121 - 17$

$= 43.5$

∴ the max profit is $43,500

e) Use vertex form!

$$P = -2(x - 5.5)^2 + 43.5$$

when I rearrange I won't switch x and P this time because I'll end up with an equation to solve for number sold (x) based on profit... get it!?

$$\pm\sqrt{\frac{P - 43.5}{-2}} = \sqrt{(x - 5.5)^2}$$

$$x - \cancel{5.5} = \pm\sqrt{-\tfrac{1}{2}(P - 43.5)} + 5.5$$
$$\cancel{+5.5}$$

$$\boxed{x = \pm\sqrt{-\tfrac{1}{2}(P - 43.5)} + 5.5}$$

f) Use the inverse equation you just found <u>or</u> plug in 31 for $P(x)$ in your profit equation.

$$x = \pm\sqrt{-\tfrac{1}{2}(31-43.5)} + 5.5$$

$$x = \pm\sqrt{6.25} + 5.5$$

$$x = 5.5 \pm 2.5$$

$x = 5.5 - 2.5$ and $5.5 + 2.5$

3 8

$\therefore$ after selling 3,000 or 8,000 items you will have made \$31,000.