What's Going On?

Checking In	Your Thoughts...
Minds on	Basic Exponent Laws
Action!	Rational Exponents
Consolidation	Big Question

Learning Goal - I will be able to simplify rational exponents.

Minds on

Basic Exponent Rules

Product Rule

When we **multiply** powers with the same base we add the exponents.

$$\underbrace{x}_{\text{base}}{}^{a} \times \underbrace{x}_{\text{base}}{}^{b} = x^{a+b}$$

$$(x^3)(x^2) = x^{3+2} = x^5$$

$$(y^{-4})(y^{-7}) = y^{-4-7} = y^{-11}$$

$$(2^6)(2^{-3}) = 2^3 = 8$$

$$(3^3)(\overset{-1}{3^{-4}})(\overset{0}{3})(3^5) = 3^{\overset{5}{}} = 243$$

Quotient Rule

When we **divide** powers with the same base we subtract the exponents.

$$\frac{x^a}{x^b} = x^{a-b}$$

$$\frac{y^3}{y^1} = y^{3-1} = y^2 \qquad \frac{\cancel{y}\cdot y\cdot y}{\cancel{y}}$$

$$\frac{2^6}{2^4} = 2^{6-4} = 2^2 = 4$$

$$\frac{3^7}{3^{-4}} = 3^{7--4} = 3^{11} = 177{,}147$$

$$\frac{(2^3)(2^{-4})}{(2^2)(2^{-7})} = \frac{2^{3-4}}{2^{2-7}} = \frac{2^{-1}}{2^{-5}}$$

$$= 2^{-1--5}$$

$$= 2^4$$

$$= 16$$

Power of a Power Rule

When we have a power raised to an exponent we multiply the exponents.

$$\left(x^a\right)^b = x^{a\times b} = x^{ab}$$

$$(xy)^n = x^n y^n$$

$$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$$

$$\left(x^3\right)^{-4} = x^{3(-4)} = x^{-12}$$

$$\left(2^2\right)^3 = 2^{(2)(3)} = 2^6 = 64$$

$$\left(3^{-3}\right)^{-1} = 3^{(-3)(-1)} = 3^3 = 27$$

$$\frac{(x^2)(x^3)^4(x^{-5})}{(x^2)^{-4}(x^3)}$$

$$= \frac{(x^2)(x^{12})(x^{-5})}{(x^{-8})(x^3)}$$

$$= \frac{x^{2+12-5}}{x^{-8+3}}$$

$$= \frac{x^9}{x^{-5}}$$

$$= x^{9--5}$$

$$= x^{14}$$

$$(x^2y^3)^4 = x^8y^{12}$$

$$(x^{-5}y^{-1})^{-3} = x^{15}y^3$$

$$\frac{(x^2y^{-2})^3}{(x^4y^{-5})^{-2}} = \frac{x^6y^{-6}}{x^{-8}y^{10}}$$

$$= x^{6--8}y^{-6-10}$$

$$= x^{14}y^{-16}$$

$$\left(\frac{x^3}{y^2}\right)^4 = \frac{x^{12}}{y^8}$$

$$\left(\frac{x^{-2}}{y^5}\right)^{-3} = \frac{x^6}{y^{-15}}$$

$$\left(\frac{(x^3y^2)(x^{-1}y^{-4})}{(x^4y^2)^3(x^{-3}y^{-1})^{-2}}\right)^3$$

$$= \left(\frac{x^{3-1}y^{2-4}}{(x^{12}y^6)(x^6y^2)}\right)^3$$

$$= \left(\frac{x^2y^{-2}}{x^{18}y^8}\right)^3$$

$$= \left(x^{2-18}y^{-2-8}\right)^3$$

$$= \left(x^{-16}y^{-10}\right)^3$$

$$= x^{-48}y^{-30}$$

Zero Exponents

Anything (except 0) raised to the exponent 0 is 1.

$$x^0 = 1 \quad , x \neq 0$$

$$\frac{\cancel{x^3}}{\cancel{x^3}} = 1 \qquad = x^{3-3} = x^0$$

Negative Exponents

A base raised to a negative exponent is equivalent to the reciprocal of the same base raised to the opposite (positive) exponent.

$$y^{-n} = \frac{1}{y^n}$$

$$\frac{1}{y^{-n}} = \frac{1}{\frac{1}{y^n}} = y^n$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$$

$$x^{-3} = \frac{1}{x^3}$$

$$2^{-5} = \frac{1}{2^5} = \frac{1}{32}$$

$$\frac{(3^3)(3^{-6})}{(3^2)(3^{-1})} = \frac{3^{-3}}{3^1}$$

$$= 3^{-3-1}$$

$$= 3^{-4}$$

$$= \frac{1}{3^4}$$

$$= \frac{1}{81}$$

$$\frac{1}{2^{-4}} = 2^4 = 16$$

$$\frac{3^1}{2^{-5}} = 3 \cdot 2^5$$

$$= 3 \cdot 32$$

$$= 96$$

$$2x^{-5}$$

$$= \frac{2}{x^5}$$

$$\frac{(2^{-4})(2^3)(2^5)}{(2^{-6})(2^2)}$$

$$= \frac{2^{-4+3+5}}{2^{-6+2}}$$

$$= \frac{2^4}{2^{-4}}$$

$$= 2^{4--4}$$

$$= 2^8$$

$$\left(\frac{3}{2}\right)^{-2} = \left(\frac{2}{3}\right)^{2} = \frac{4}{9}$$

$$\left(\frac{x^3}{y^{-1}}\right)^{-2} = \left(\frac{y^{-1}}{x^3}\right)^{2} = \frac{y^{-2}}{x^6} = \frac{1}{x^6 y^2}$$

$$\left(\frac{(x^3y^2)}{(x^4)^{-2}}\right)^{-1} = \left(\frac{(x^4)^{-2}}{(x^3y^2)}\right)$$

$$= \frac{x^{-8}}{x^3y^2}$$

$$= \frac{x^{-8-3}}{y^2}$$

$$= \frac{x^{-11}}{y^2}$$

$$= \frac{1}{x^{11}y^2}$$

Action!

Rational Exponents

Rational Exponents

Anything raised to a rational exponent is a radical.

The rational exponent $\frac{1}{n}$ indicates the nth root of the base.

$$x^{\frac{1}{n}} = \sqrt[n]{x} \quad ,n > 1, n \in R, x \neq 0$$

$$x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$, m \in Z, m > 0, n \in Z, n > 0, x \neq 0$$

$\sqrt[3]{216} = 6$

$\sqrt[5]{16807} = 7$

$\sqrt[5]{-1024} = -4$

$\boxed{\begin{array}{l} \sqrt{9} = \pm 3 \\ \sqrt[4]{16} = \pm 2 \end{array}}$

$81^{\frac{1}{4}} = \sqrt[4]{81} = \pm 3$

$125^{\frac{1}{3}} = \sqrt[3]{125} = 5$

$$16^{\frac{3}{2}} = \sqrt{16^3} \text{ or } \sqrt{16}^3 = 64$$

$$81^{\frac{3}{4}} = \sqrt[4]{81^3} \text{ or } \left(\sqrt[4]{81}\right)^3 = 27$$

$$243^{\frac{2}{5}} = \sqrt[5]{243^2} \text{ or } \left(\sqrt[5]{243}\right)^2$$

$$= 9$$

Consolidation

Big Question!

Evaluate

$$81^{\frac{1}{2}} + \sqrt[3]{8} - 32^{\frac{4}{5}} + 16^{\frac{3}{4}}$$

$$= \sqrt{81} + \sqrt[3]{8} - (\sqrt[5]{32})^4 + (\sqrt[4]{16})^3$$

$$= 9 + 2 - (2)^4 + (2)^3$$

$$= 11 - 16 + 8$$

$$= -5 + 8$$

$$= 3$$

Write as a single power.
Then Evaluate

$$\frac{(8^{-2})(8^{2.5})}{(8^{6})^{-0.25}}$$

$$= \frac{8^{-2+2.5}}{8^{6(-0.25)}}$$

$$= \frac{8^{0.5}}{8^{-1.5}}$$

$$= 8^{0.5--1.5}$$

$$= 8^{2}$$

$$= 64$$

or

$$\frac{(8^{-2})(8^{\frac{5}{2}})}{(8^{6})^{-\frac{1}{4}}}$$

I prefer leaving the decimals until the end for obvious reasons