Introduction to Sequences and Series

Arithmetic and Geometric Sequences

Definitions

Sequence: An ordered list of numbers.

Examples: 635, 630, 625, 620, 615 …

128, 64, 32, 16, 8 …

Term: A number in a sequence. Subscripts are usually used to identify the positions of the terms.

Examples: $t_1 = 635$, $t_2 = 630$, $t_3 = 625$ …

Examples: $t_1 = 128$, $t_2 = 64$, $t_3 = 32$ …

Arithmetic Sequence: A sequence that has the same difference, the **common difference**, between any pair of consecutive terms.

Examples: 6, 8, 10, 12, 14 …

150, 141, 132, 123, 114 …

Geometric Series: A sequence that has the same ratio, **common ratio**, between any pair of consecutive terms.

Examples: 4, 8, 16, 32, 64 …

2000, 1000, 500, 250, 125 …

Recursive Sequence: A sequence for which one term (or more) is given and each successive term is determined from the previous term(s).

Examples: $6, 6 + 2, (6 + 2) + 2, (6 + 2 + 2) +2$ …

$a, a + d, (a + d) + d, (a + d + d) + d$ …

$4, 4 \times 2, (4 \times 2) \times 2, (4 \times 2 \times 2) \times 2$ …

$a, a \times r, (a \times r) \times r, (a \times r \times r) \times r$ …

General Term: A formula, labelled t_n, that expresses each term of a sequence as a function of its position. For example, if the general term is $t_n = 2n$, then to calculate the 12^{th} term (t_{12}), substitute $n = 12$.

Examples: $t_n = 2n + 4$ OR $t_n = 6 + (n – 1)2$

$t_n = a + (n – 1)d$, a represents the first term, d represents the difference between successive terms

$t_n = 4 \times 2^{n-1}$

$t_n = a \times r^{n-1}$, a represents the first term, r represents the ratio of successive terms

Recursive Formula: A formula relating the general term of a sequence to the previous term(s).

Examples: $t_1 = 6$, $t_n = t_{n-1} + 2$, where $n > 1$

$t_1 = a$, $t_n = t_{n-1} + d$, where $n > 1$

$t_1 = 4$, $t_n = 4 \times t_{n-1}$, where $n > 1$

$t_1 = a$, $t_n = rt_{n-1}$, where $n > 1$

Worked Examples

1. Decide whether the given sequence is algebraic or geometric. Then determine the general term and the recursive formula for the sequence. Use your formulae to determine the 10^{th} term of the sequence.

a. 6, 15, 24, 33, 42 …

The terms in the sequence <u>increase by 9</u>, therefore this is an **arithmetic sequence**.

The formula for the general term of an arithmetic sequence is $\mathbf{t_n = a + (n – 1)d}$.
For this example, a = 6 and d = 9.
Therefore, the general term for this example is $\mathbf{t_n = 6 + (n – 1)9}$ OR $\mathbf{t_n = 9n – 3}$

The generalized recursive formula for an arithmetic sequence is $t_1 = a$, $t_n = t_{n-1} + d$, where $n > 1$.
Therefore, the recursive formula for this example is $\mathbf{t_1 = 6, t_n = t_{n-1} + 9}$**, where** $\mathbf{n > 1}$.

The 10^{th} term in this sequence is $\mathbf{t_n = 6 + ((10) – 1)9 = 6 + (9)9 = 6 + 81 = \underline{87}}$.

b. 3, -12, 48, -192, 768 …
The terms in the sequence are in a <u>ratio</u> of -4. Therefore this is an **geometric sequence**.

The formula for the general term of a geometric sequence is $\mathbf{t_n = a \times r^{n-1}}$.
For this example, a = 3 and r = -4.
Therefore, the general term for this example is $\mathbf{t_n = 3 \times (-4)^{n-1}}$.

The generalized recursive formula for a geometric sequence is $\mathbf{t_1 = a, t_n = rt_{n-1}}$**, where** $\mathbf{n > 1}$.
Therefore, the recursive formula for this example is $\mathbf{t_1 = 3, t_n = (-4)t_{n-1}}$**, where** $\mathbf{n > 1}$.

The 10^{th} term in this sequence is $\mathbf{t_n = 3 \times (-4)^{10-1} = 3 \times (-4)^9 = 3 \times -262{,}144 = \underline{-786{,}432}}$

Questions

Complete the table for each sequence given below.

$n > 1$

Sequence	Arithmetic or Geometric	Difference (d) or Ratio (r)	First Term (a)	General Term (t_n)	Recursive Formula (t_1, t_n)	13^{th} Term
1, 5, 9, 13, 17 …	A	$d=+4$	1	$t_n=1+(n-1)4$	$t_1=1, t_n=t_{n-1}+4$	49
3, 6, 12, 24, 48 …	G	$r=2$	3	$t_n=3\times 2^{n-1}$	$t_1=3, t_n=t_{n-1}\times 2$	12288
8, 11, 14, 17, 20 …	A	$d=3$	8	$t_n=8+3(n-1)$	$t_1=8, t_n=t_{n-1}+3$	44
28, 19, 10, 1, -8 …	A	$d=-9$	28	$t_n=28+(n-1)-9$	$t_1=28, t_n=t_{n-1}-9$	-80
5, 15, 45, 135 …	G	$r=3$	5	$t_n=5\times 3^{n-1}$	$t_1=5, t_n=3t_{n-1}$	2657205
10 125, 6 750, 4 500 …	G	$r=\frac{2}{3}$	10125	$t_n=10125\times\left(\frac{2}{3}\right)^{n-1}$	$t_1=10125, t_n=\left(\frac{2}{3}\right)t_{n-1}$	78.04
125, 50, 20, 8 …	G	$r=0.4$	125	$t_n=125\times 0.4^{n-1}$	$t_1=125, t_n=0.4t_{n-1}$	0.002
784, 588, 392 …	A	$d=-196$	784	$t_n=784+(n-1)-196$	$t_1=784, t_n=t_{n-1}-196$	-1566
4, 20, 100 …	G	$r=5$	4	$t_n=4\times 5^{n-1}$	$t_1=4, t_n=5t_{n-1}$	976,562,500
15, -60, 240 …	G	$r=-4$	15	$t_n=15\times(-4)^{n-1}$	$t_1=15, t_n=-4t_{n-1}$	251,658,240