MCR 3U Review Questions: Sequences and Series

1.

For each sequence, determine:	$\times 3 \quad \times 3$ a) 4, 12, 36, …	$-11 \quad -11$ b) 31, 20, 9, …
The general term.	$t_n = ar^{n-1}$ $t_n = 4 \times 3^{n-1}$	$t_n = a + (n-1)d$ $t_n = 31 + (n-1)(-11)$
The recursive formula.	$t_1 = 4, t_n = 3t_{n-1}$	$t_1 = 31, t_n = t_{n-1} - 11$
t_{15} (15th term)	$t_{15} = 4 \times 3^{14}$ $= 4 \times 4782969$ $= 19{,}131{,}876$	$t_{15} = 31 + (14)(-11)$ $= 31 - 154$ $= -123$
s_{15} (sum of the first 15 terms)	$S_{15} = \frac{4(3^{15} - 1)}{3 - 1}$ $= 28{,}697{,}812$ *could have used other formula by finding t_{16} $(3 \times t_{15})$	$S_{15} = \frac{15(31 - 123)}{2}$ $= -690$

2. An opera house has 27 seats in the first row, 34 seats in the second row, 41 seats in the third row, and so on. The last row has 181 seats..

arithmetic

   a. How many seats are in the 10^{th} row?

$a / t_1 = 27$

$d = 7$

$t_n = 181$

$$t_{10} = 27 + (9)7$$
$$= 27 + 63$$
$$= 90 \text{ seats in Row 10!}$$

   b. How many rows of seats are in the opera house?

Find n

$$t_n = a + (n-1)d$$
$$181 = 27 + (n-1)7$$
$$181 = 27 + 7n - 7$$
$$181 = 20 + 7n$$
$$-20 \quad -20$$
$$161 = 7n$$
$$n = 23$$

∴ there are 23 rows

3. Guy purchased a rare stamp for \$820 in 2001. If the value of the stamp increases by 10% per year, how much will the stamp be worth in 2010?

820

902

×1.10

$r = 1.1$

$$t_{10} = 620 \times 1.1^9$$
$$= 1{,}933.52$$

4. Calculate the sum of each series
   a. 123 + 118 + 113 + … − 122

$-5 \quad -5$

*arithmetic!

We need n!!

$a/t_1 = 123$

$d = -5$

$t_n = -122$

$t_n = a + (n-1)d$

$-122 = 123 + (n-1)(-5)$

$-122 = 123 - 5n + 5$

$-122 = 128 - 5n$

$-128 \quad -128$

$\frac{-250}{-5} = \frac{-5n}{-5}$

$n = 50$

$S_n = \frac{50(123-122)}{2}$

$\boxed{S_n = 25}$

   b. $1 + \frac{5}{2} + \frac{25}{4} + \cdots + \frac{15625}{64}$

*geometric!

$r = \frac{5}{2}$ or 2.5

$a/t_1 = 1$

$t_n = \frac{15625}{64}$

Find t_{n+1}

$t_{n+1} = \frac{5}{2} \times \frac{15625}{64} = \boxed{\frac{78125}{128}}$

$r \times t_n$

$S_n = \frac{\frac{78125}{128} - 1}{\frac{5}{2} - 1}$

$S_n = \frac{\frac{77997}{128}}{\frac{3}{2}} = \frac{77997}{384}$

$\frac{155994}{384}$

5. Determine the 100^{th} term of the given sequence. Explain your reasoning.

$$\frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{4}{11}, \ldots$$

0.5, 0.4, 0.375

neither arithmetic nor geometric

numerator ⇒ n

denominator ⇒ adding 3 each time ⇒ a=2, d=3

$t_n = 2 + (n-1)3$

$= 2 + 3n - 3$

$= 3n - 1$

$t_n = \frac{n}{3n-1}$

$t_{100} = \frac{100}{299}$

6. The 7th term of a geometric sequence is 3 and the 11th term is 48.

a. Determine the 37th term without finding the general term.

$t_7 = 3$

$t_{11} = 48$

$\frac{t_{11}}{t_7} = \frac{48}{3} = 16$

-from t_7 to t_{11}, we multiplied by same number four times

$\sqrt[4]{16}$

$t_{37} = 3 \times 2^{30}$

$t_{37} = 3{,}221{,}225{,}472$

b. Determine the general term.

first find a

$a = \frac{3}{2^6} = 0.046875$

$t_n = 0.046875 \times 2^{n-1}$

c. Determine the sum of the first 29 terms.

$$S_{29} = \frac{0.046875(2^{29}-1)}{2-1} = 25\,165\,823.95$$

7. The 7th term in an arithmetic sequence is 465 and the 13th term is 219.

a. Determine the 100th term without finding the general term.

93 terms from 7th to 100th

$219 - 465 = -246$

$\frac{-246}{6} = -41$ (d)

← 6 terms btw 7th and 13th

$t_{100} = 465 + (-41)(93)$

7th term

$= -3348$

b. Determine the general term.

Find a

add 41 to 465

6 times ⇒ $a = 711$

$t_n = 711 + (n-1)(-41)$

c. Determine the sum of the first 100 terms.

$$S_{100} = \frac{100[2(711) + (99)(-41)]}{2}$$

$S_{100} = -131{,}850$