New Unit

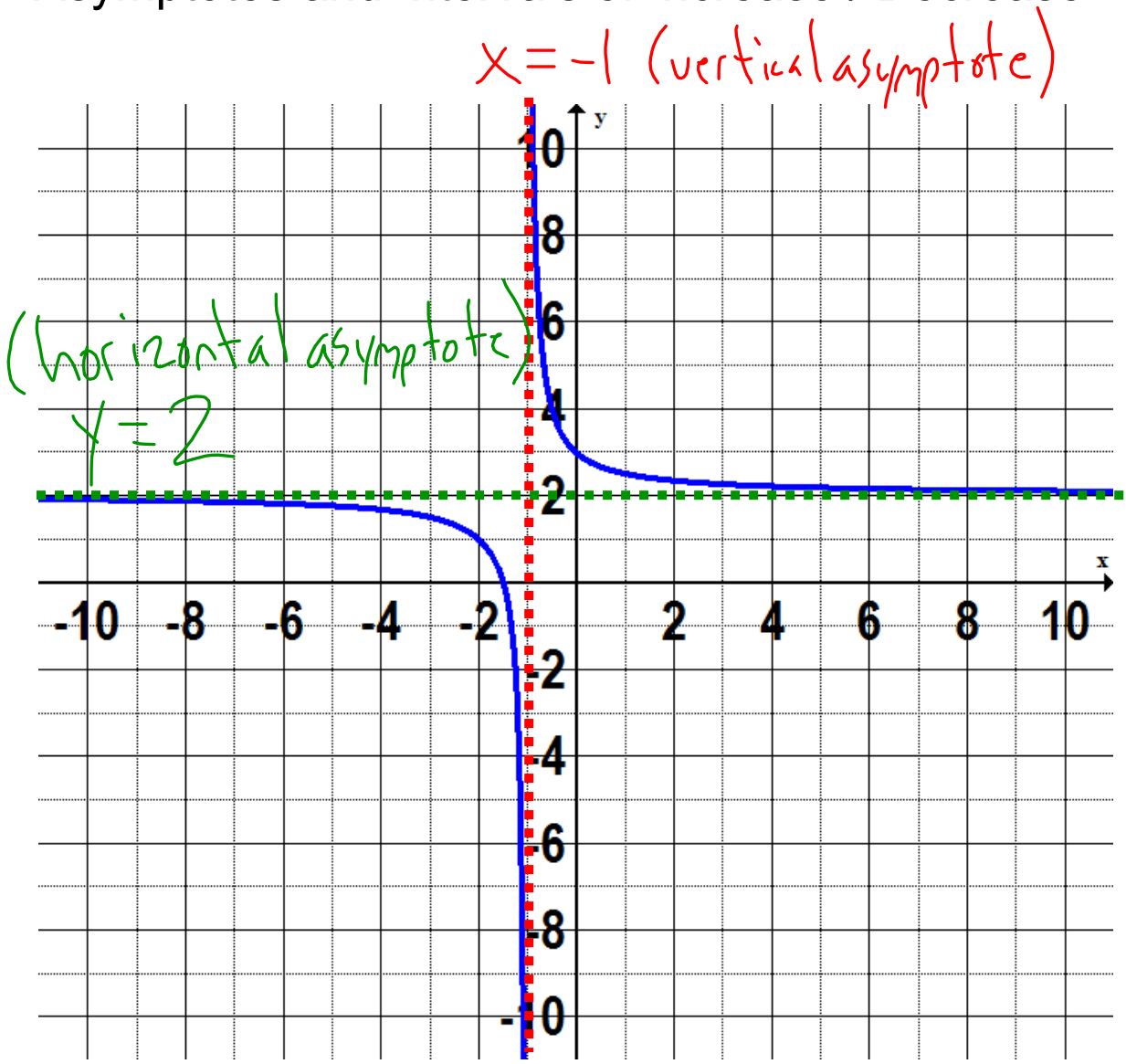
Rational Functions, Equations and Inequalities

Minds On

Asymptotes and Intervals of Increase / Decrease

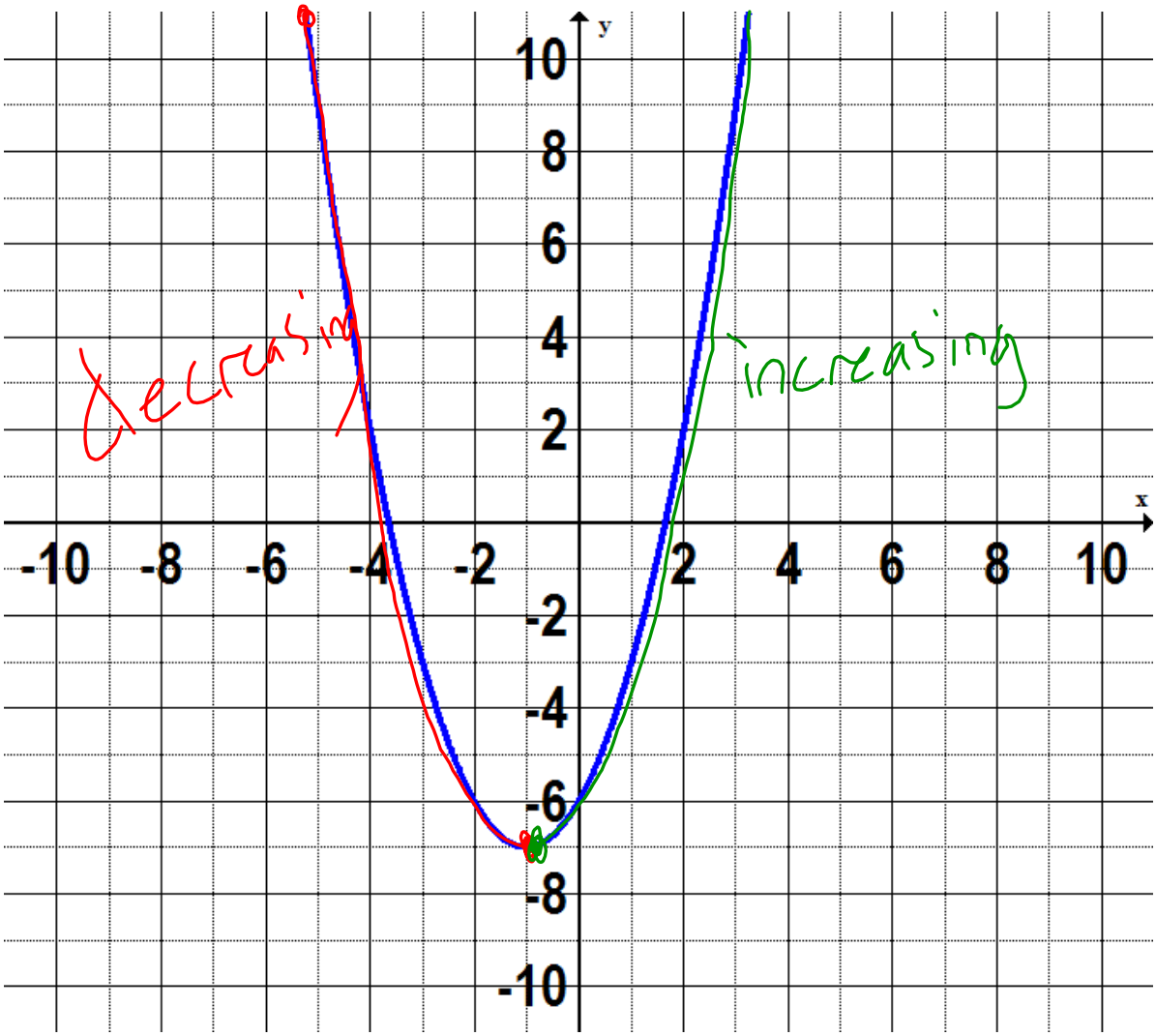
Minds On

Asymptotes and Intervals of Increase / Decrease



Minds On

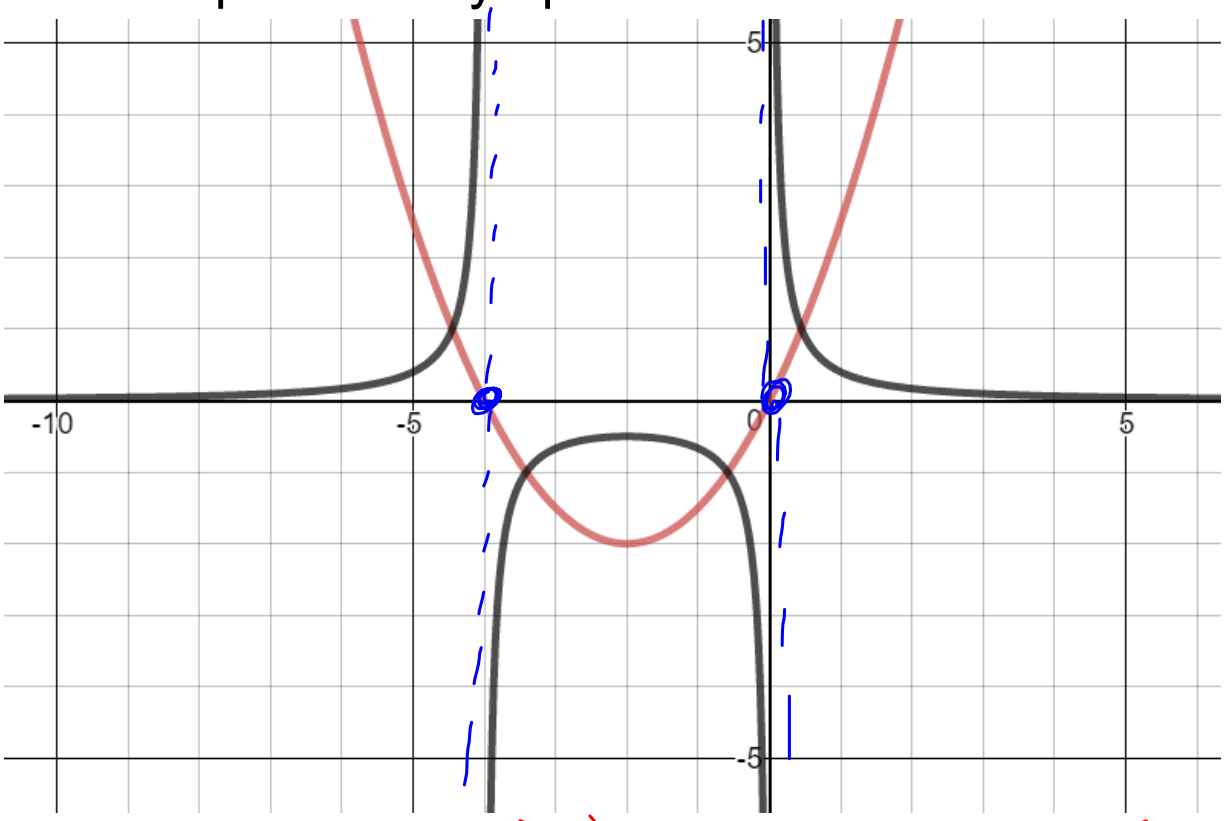
Asymptotes and Intervals of Increase / Decrease



Action

Desmos Investigation

x-intercepts and asymptotes



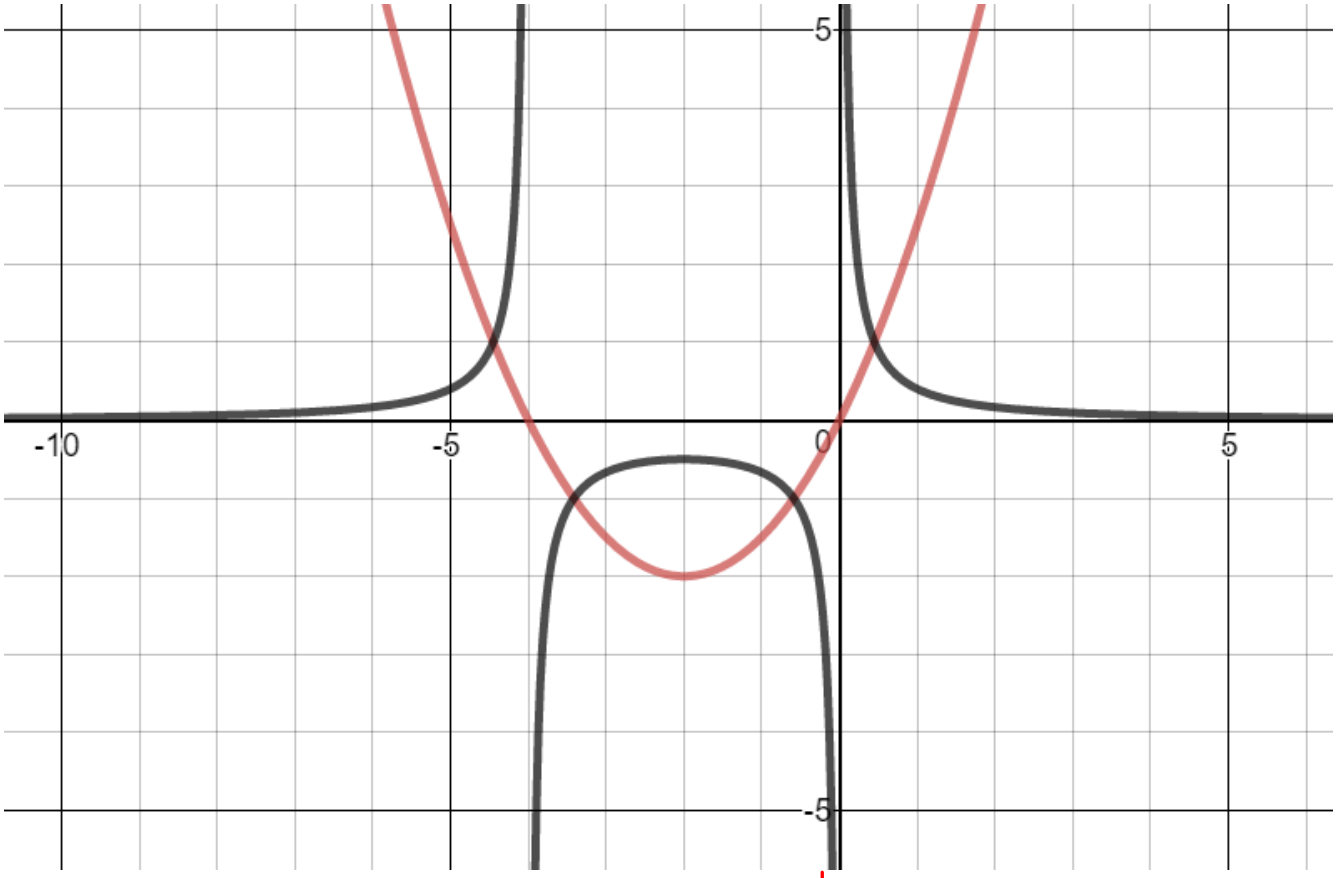
Where $f(x)$ has zeroes $\frac{1}{f(x)}$ has vertical asymptotes.

$\frac{1}{f(x)}$ always has a horizontal asymptote when $y=0$

Action

Desmos Investigation

intervals of increase and decrease



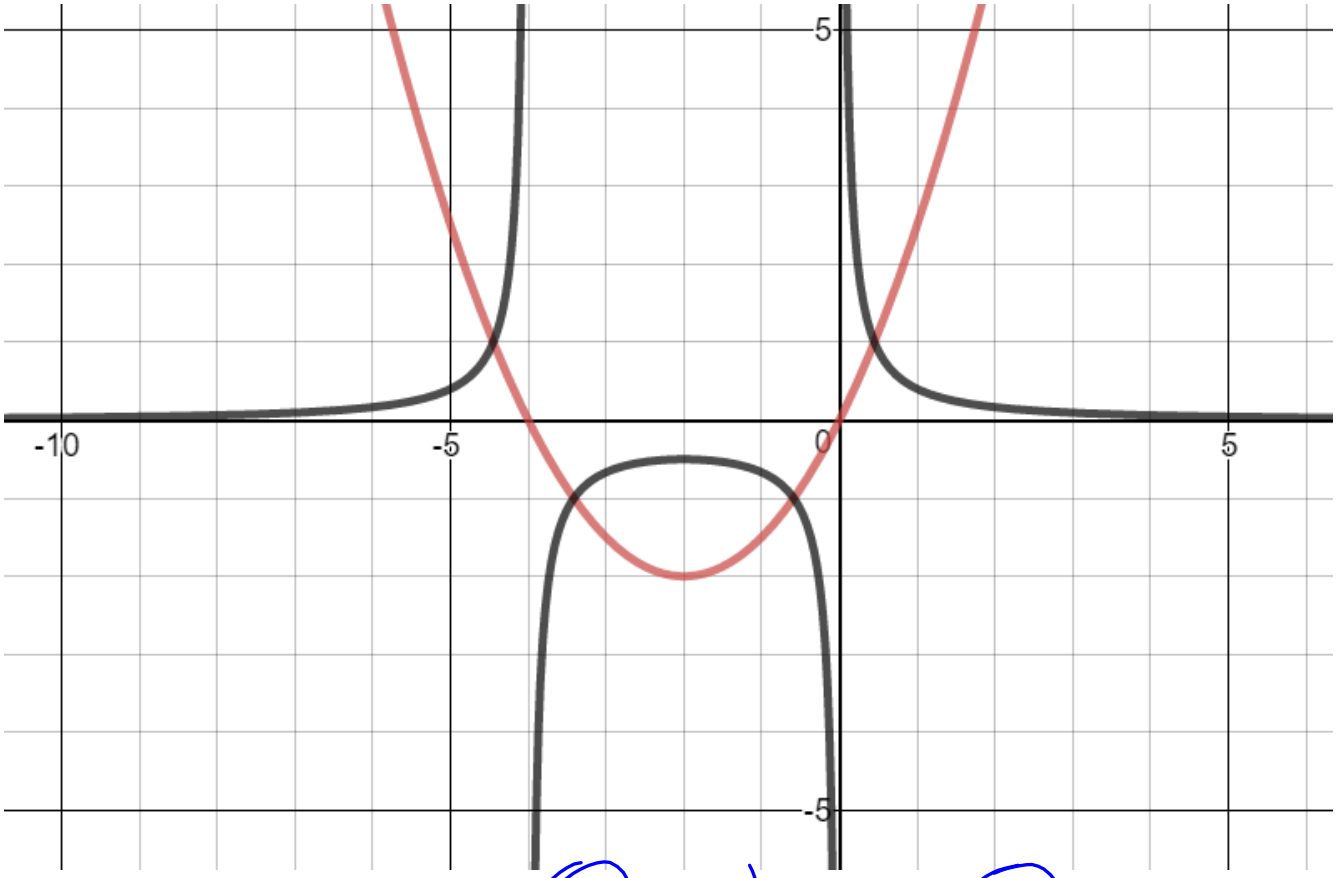
When $f(x)\uparrow$, $\frac{1}{f(x)}\downarrow$

and vice versa

Action

Desmos Investigation

intervals of positive and negative

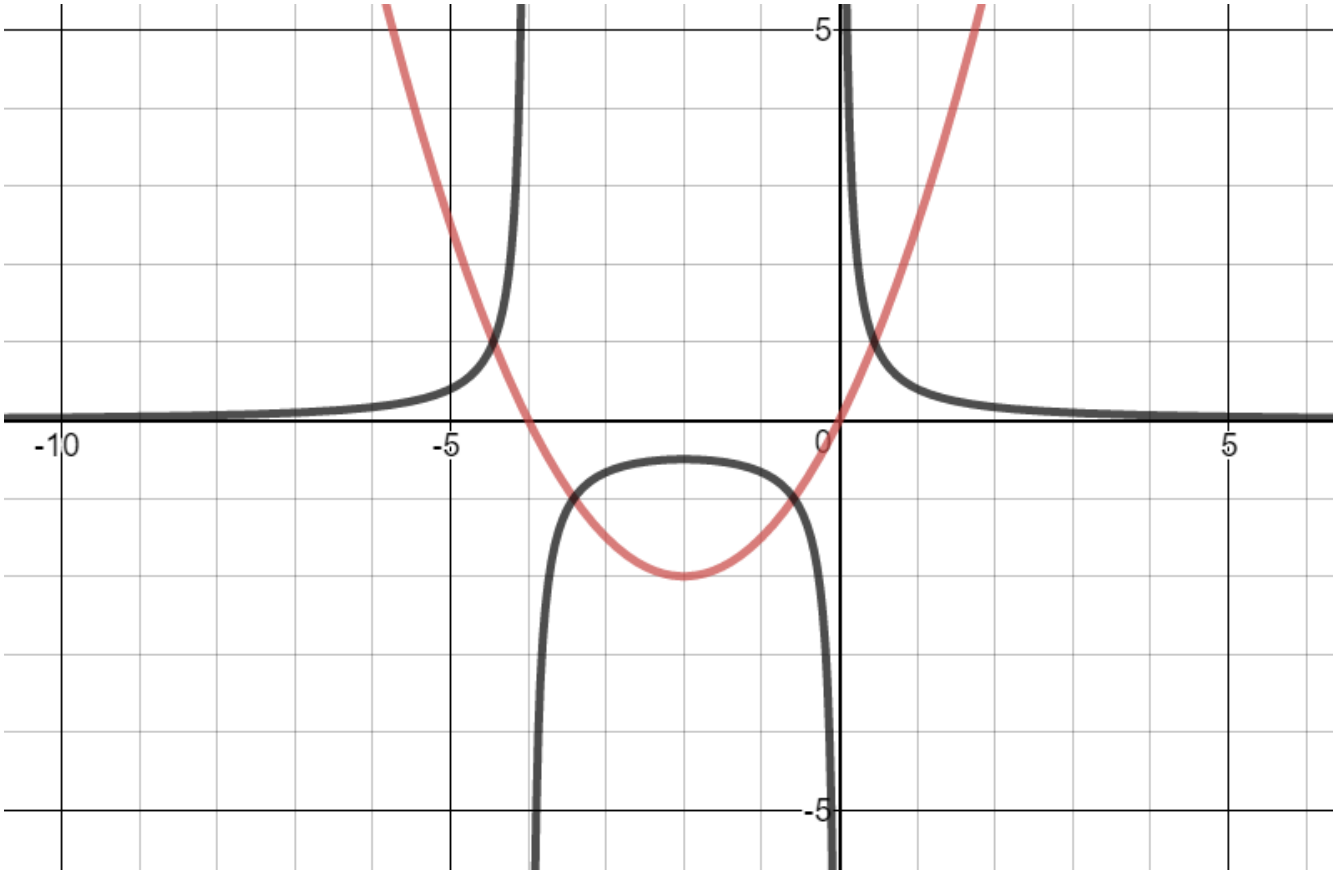


When $f(x)$ ⊕ $\frac{1}{f(x)}$ ⊕

Action

Desmos Investigation

points of intersection

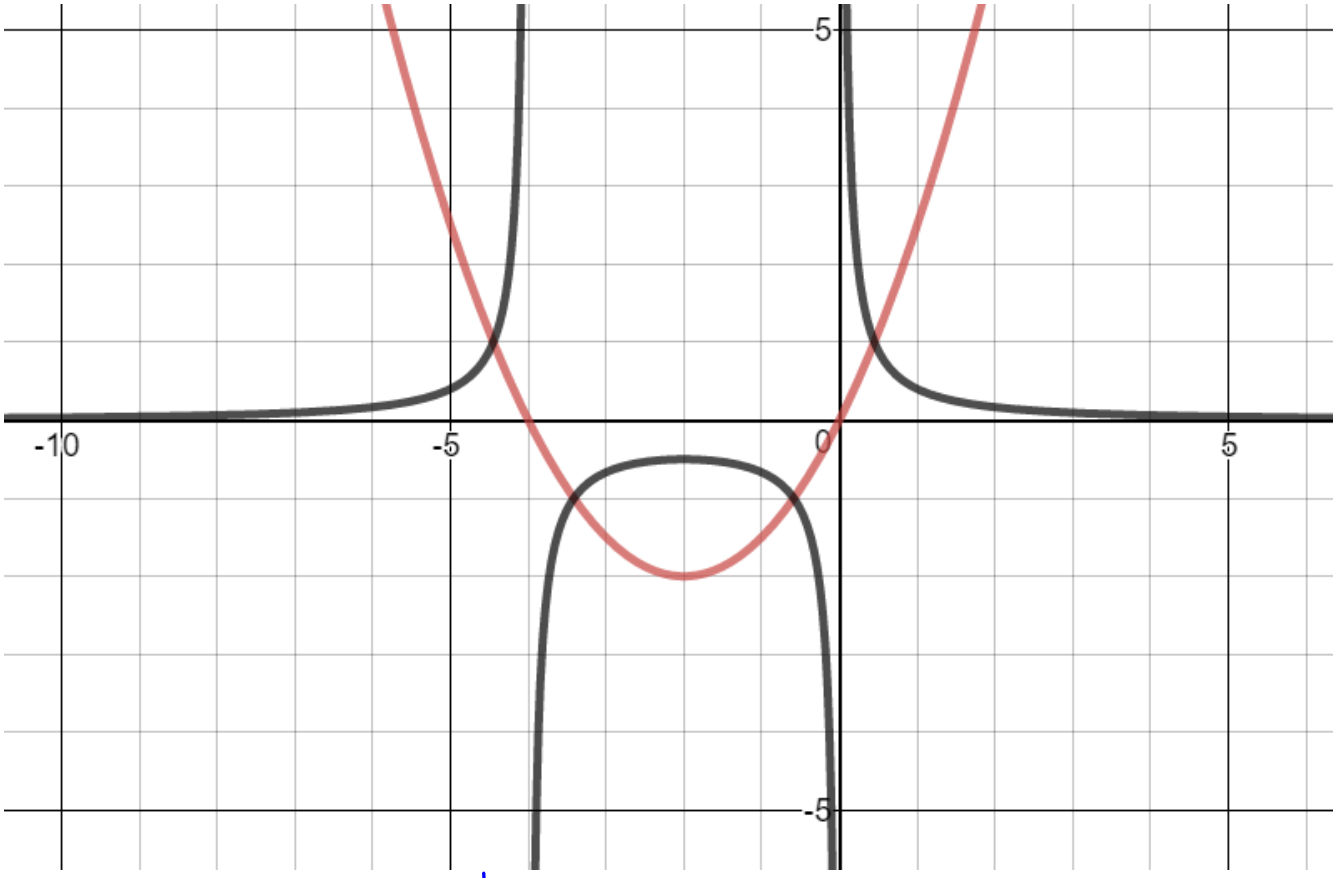


y-values of POI are always 1 and -1

Action

Desmos Investigation

max and min values



When $f(x)$ has a max, $\frac{1}{f(x)}$ has a min

Action

All the y coordinates of a reciprocal function are the reciprocal of the y-coordinates of the original function ($\frac{1}{\text{y-coord}}$)

The graph of a reciprocal function has a vertical asymptote at each zero of the original function.

A reciprocal function will always have y = 0 as a horizontal asymptote if the original function is linear or quadratic.

A reciprocal function has the same positive/negative intervals as the original function

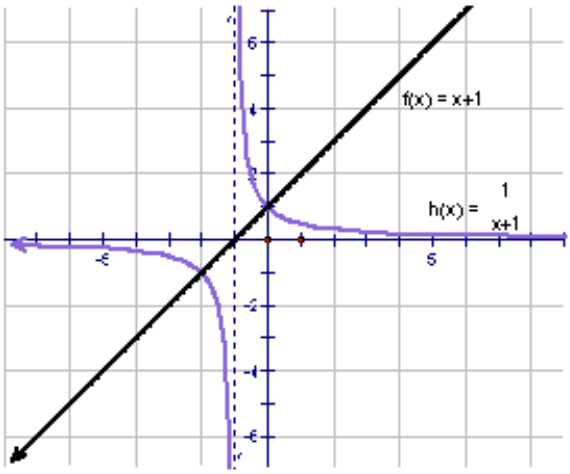
Intervals of increase on the original function are intervals of decrease on the reciprocal function. Intervals of decrease on the original function are intervals of increase on the reciprocal function.

If the range of the original function includes 1 and/or -1, the reciprocal function will intersect the original function at a point (or points) where the y-coordinate is 1 or -1.

If the original function has a local minimum point, the reciprocal function will have a local maximum point at the same x-value (and vice versa)

Action

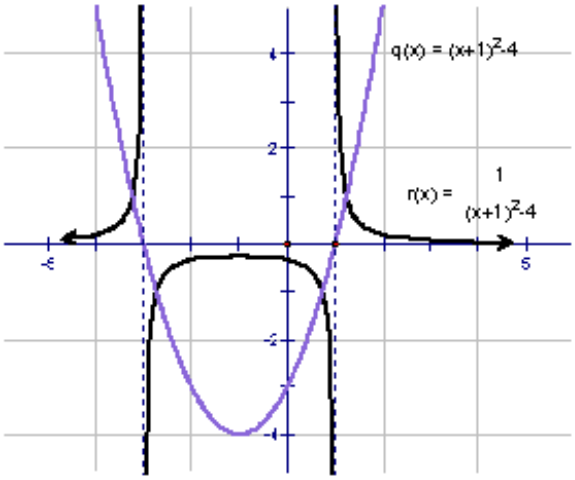
A linear function and its reciprocal



Both functions are negative when $x \in (-\infty,-1)$ and positive when $x \in (-1, \infty)$. The original function is increasing when $x \in (-\infty, \infty)$. The reciprocal function is decreasing when $x \in (-\infty, -1)$ or $(-1, \infty)$

Action

A quadratic function and it's reciprocal



Both functions are negative when x ∈ (-3,1) and positive when x ∈ (-∞, -3) or (1, ∞). The original function is decreasing when x ∈ (-∞,-1) and increasing when x ∈ (-1, ∞). The reciprocal function is increasing when x ∈ (-∞, -3) or (-3, -1) and decreasing when x ∈ (-1, 1) or (1, ∞).

Action

Example 1

Given the function $f(x) = 2 - x$ $f(x) = -x + 2$

a) Determine the domain and range, intercepts, positive/negative intervals, and increasing/decreasing intervals

b) Use your answers for part a) to sketch the graph of the reciprocal function

$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R}\}$

x-int = 2

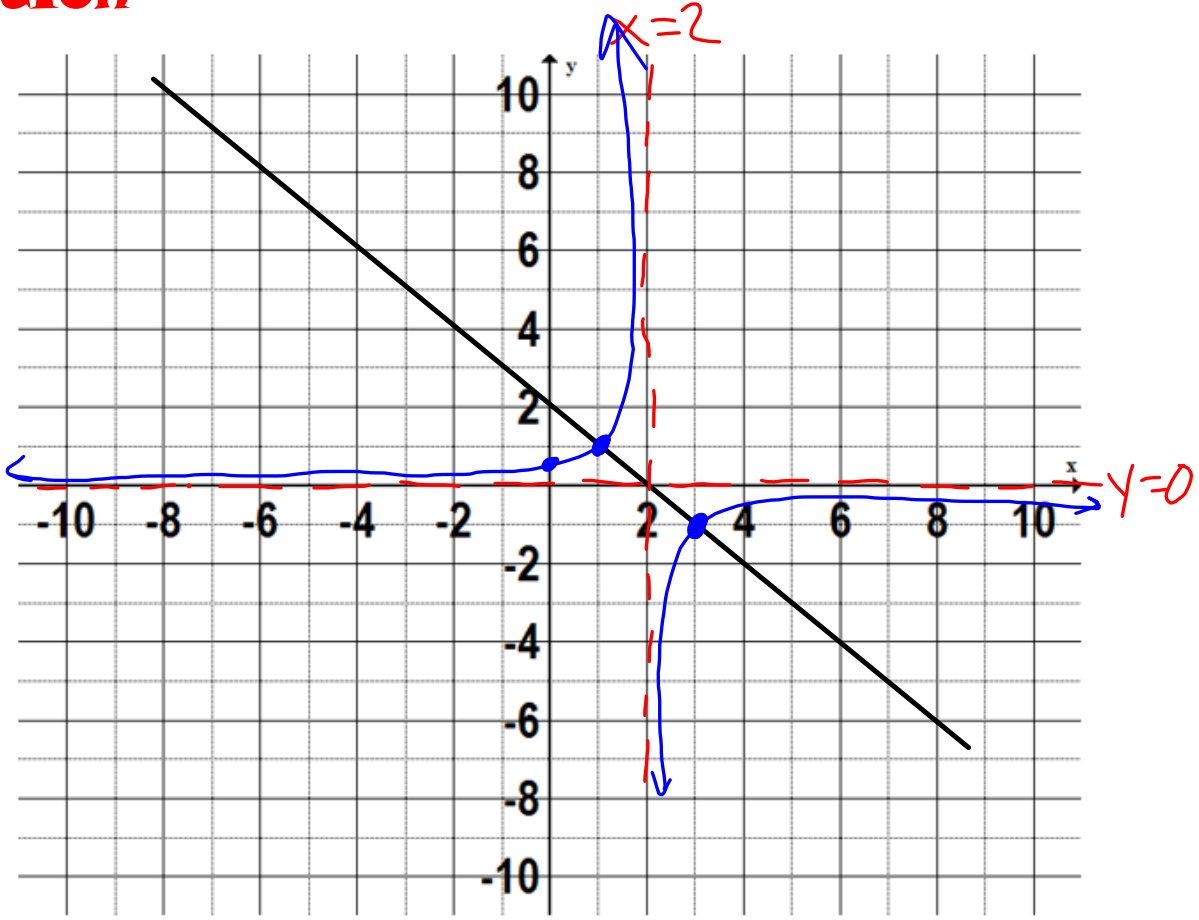
y-int = 2

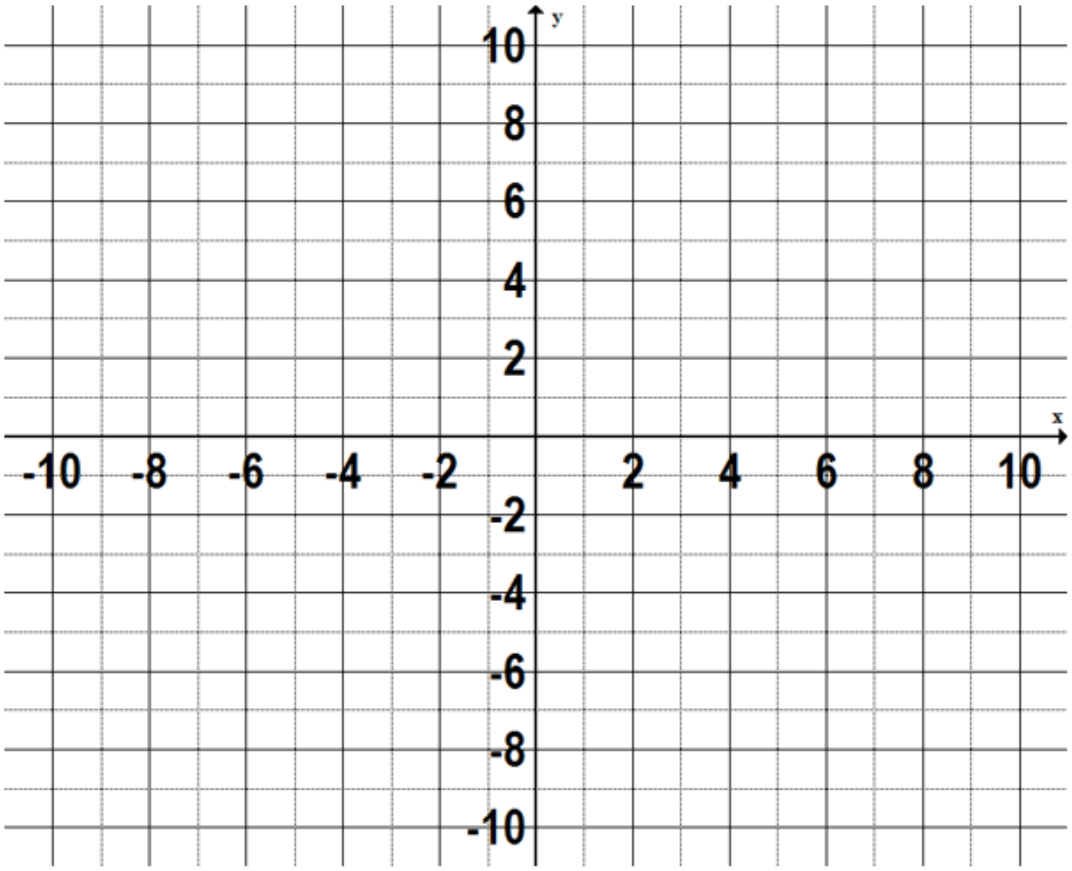
⊕ when $x \in (-\infty, 2)$

⊖ when $x \in (2, +\infty)$

↓ when $x \in (-\infty, +\infty)$

Action


x=2
y=0
x
y
10
8
6
4
2
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
2
4
6
8
10


x
y
10
8
6
4
2
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
2
4
6
8
10

Action

Example 2

Given the function $f(x) = 9 - x^2$ $f(x) = -x^2 + 9$

a) Determine the domain and range, intercepts, positive/negative intervals, and increasing/decreasing intervals

b) Use your answers for part a) to sketch the graph of the reciprocal function

$D = \{x \in \mathbb{R}\}$

$R = \{f(x) \in \mathbb{R} \mid f(x) \leq 9\}$

$f(x) = -(x^2 - 9)$

$f(x) = -(x+3)(x-3)$

y-int = (0,9)

x-int = (3,0), (-3,0)

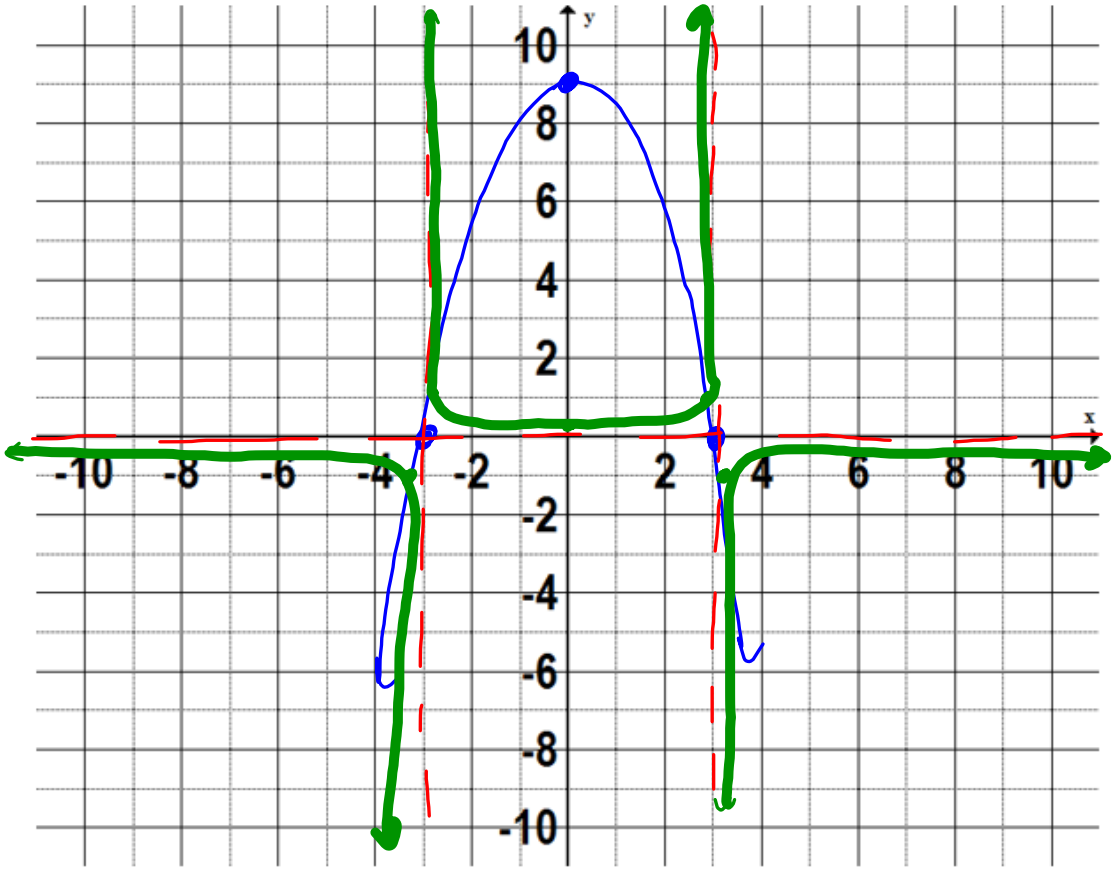
⊕ when $x \in (-3, 3)$

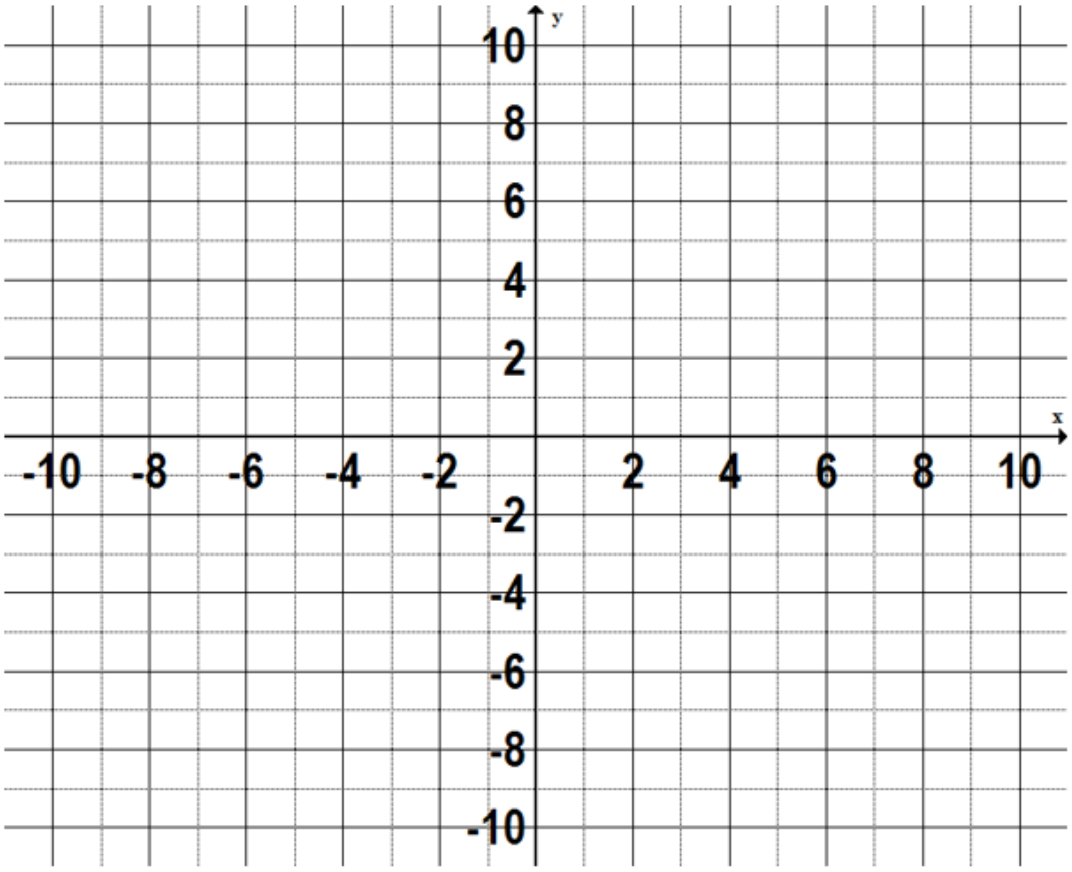
⊖ when $x \in (-\infty, -3), (3, +\infty)$

↑ when $x \in (-\infty, 0)$

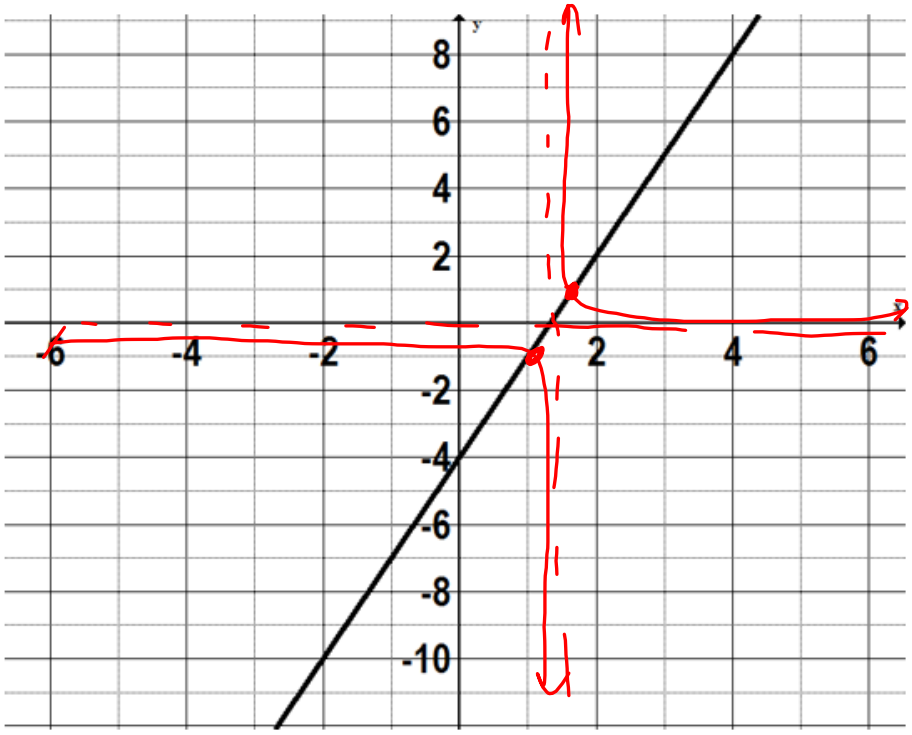
↓ when $x \in (0, +\infty)$

Action

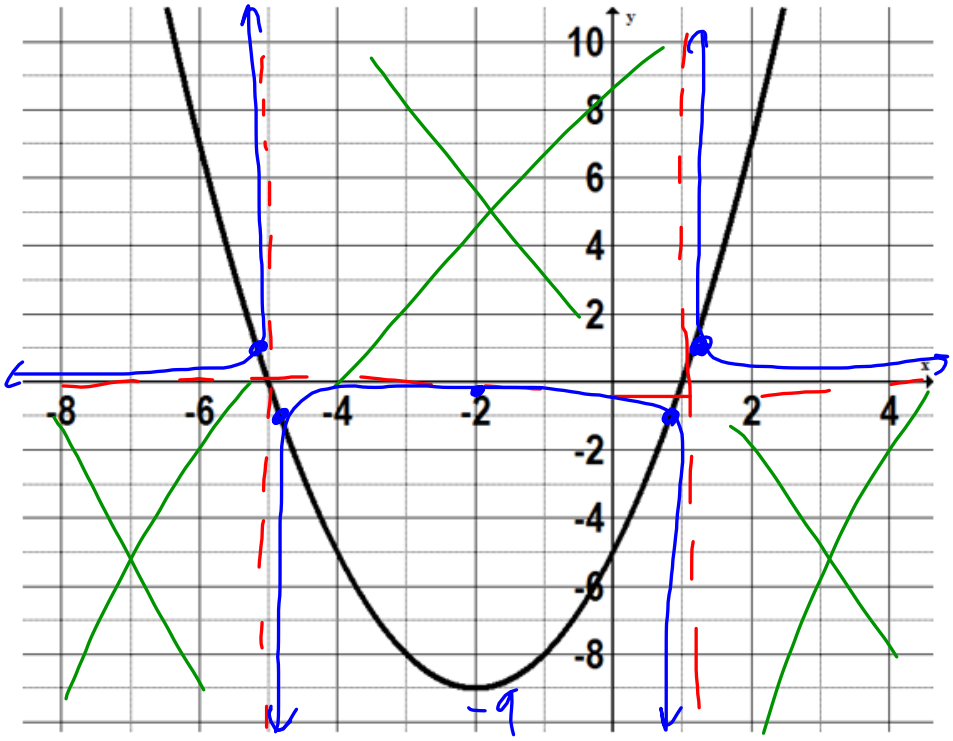

y
x
10
8
6
4
2
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
2
4
6
8
10


y
x
10
8
6
4
2
-2
-4
-6
-8
-10
-10
-8
-6
-4
-2
2
4
6
8
10

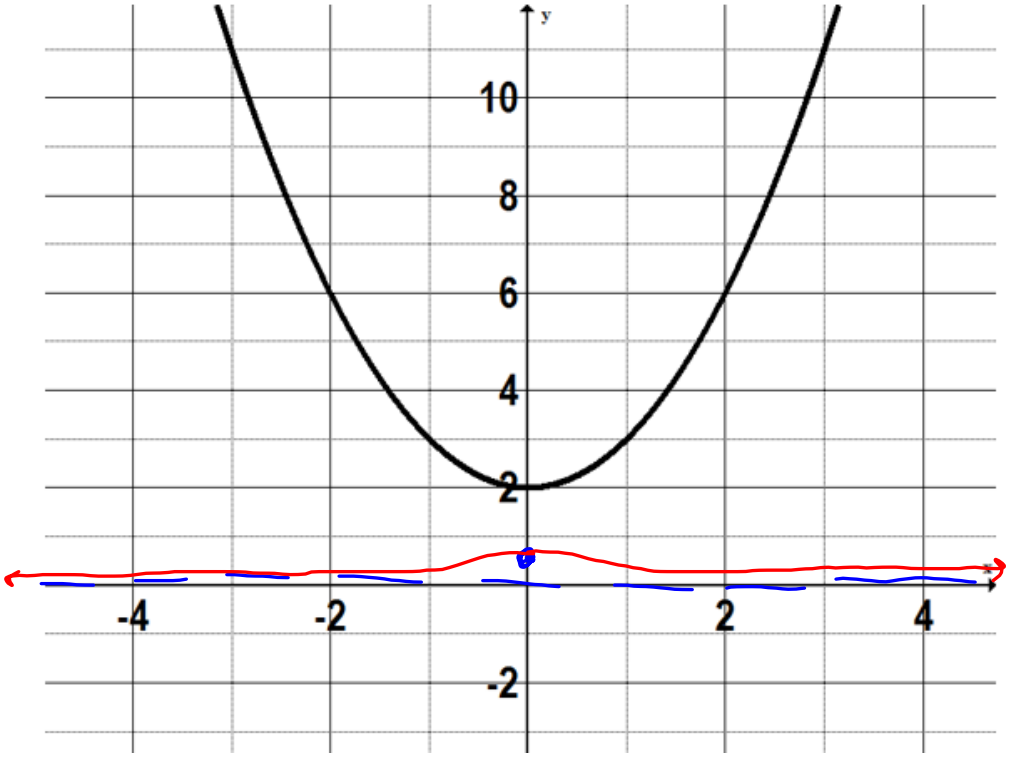
The graph of $f(x) = 3x - 4$ is shown below. Sketch the graph of its reciprocal function.



The graph of $f(x) = (x - 1)(x + 5)$ is shown below. Sketch the graph of its reciprocal function.



The graph of $f(x) = x^2 + 2$ is shown below. Sketch the graph of its reciprocal function.



Consolidation

page 254 #1-3, 5-8 (some), 9