Learning Goal: I will be able explore graphs that are created by dividing polynomial functions

Minds On: Investigation on page #258 (in the Nelson textbook - organizer to accompany)

Action: Class Note

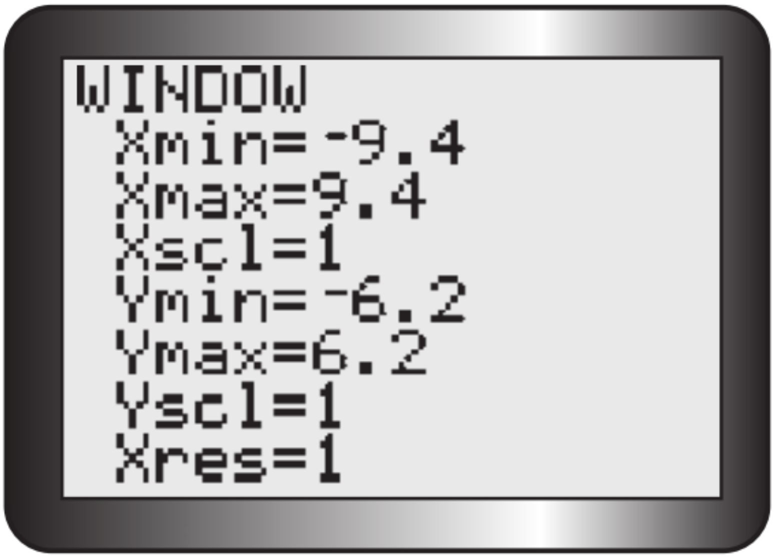
Consolidation: page 262 #1-3 + name that asymptote!

We will be skipping RAFT today. Sorry.

You can read at the end of class if there is time.

Minds On

TI-83 Investigation



Minds On

Function	Graph	Holes?	Asymptotes?
$\frac{x+1}{1-x^2}$		$x=-1$	$x=1$ $y=0$
$\frac{1-x^2}{x+1}$		$x=-1$	

Function	Graph	Holes?	Asymptotes?
$\frac{x}{x-1}$		X	$x=1$ $y=1$
$\frac{x-1}{x}$		X	$x=0$ $y=1$

Function	Graph	Holes?	Asymptotes?
$\frac{x^2-1}{x-1}$		$x=1$	
$\frac{x-1}{x^2-1}$		$x=1$	$x=-1$ $y=0$

Function	Graph	Holes?	Asymptotes?
$\frac{x^2+2x}{x+1}$		X	$x=-1$ weird one
$\frac{x+1}{x^2+2x}$	$y=\frac{x+1}{x^2+2x}$ $0=\frac{x+1}{x^2+2x}$	X	$x=0$ $x=-2$ $y=0$

$0=x+1$

Function	Graph	Holes?	Asymptotes?
$\frac{x^2 - 2x - 3}{x + 1}$		$x = -1$	
$\frac{x + 1}{x^2 - 2x - 3}$		$x = -1$	$x = 3$ $y = 0$

Function	Graph	Holes?	Asymptotes?
$\frac{2x^2-3}{x^2+1}$			$y=2$
$\frac{x^2+1}{2x^2-3}$	$2x^2-3=0$ $2x^2=3$ $\sqrt{x^2}=\sqrt{1.5}$ $x=\pm\sqrt{1.5}$		$x=\pm\sqrt{1.5}$ $y=0.5$

Function	Graph	Holes?	Asymptotes?
$\frac{0.5x^2+1}{x-1}$		X	$x=1$ a weird one

Action

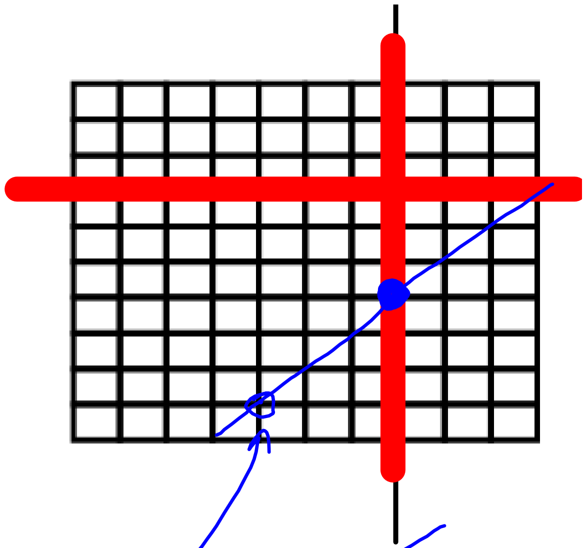
Key Ideas

- The quotient of two polynomial functions results in a rational function which often has one or more discontinuities.
- The breaks or discontinuities in a rational function occur where the function is undefined. The function is undefined at values where the denominator is equal to zero. As a result, these values must be restricted from the domain of the function.
- The values that must be restricted from the domain of a rational function result in key characteristics that define the shape of the graph. These characteristics include a combination of vertical asymptotes (also called infinite discontinuities) and holes (also called point discontinuities).
- The end behaviours of many rational functions are determined by either horizontal asymptotes or oblique asymptotes.

Action

Let's look at some concrete examples:

A rational function, $f(x) = p(x)/q(x)$, has a hole at $x = a$ if $p(a)/q(a) = 0/0$. This occurs when $p(x)$ and $q(x)$ contain a common factor of $(x - a)$.



Example: $\frac{x^2-9}{x+3}$

$$= \frac{\cancel{(x+3)}(x-3)}{\cancel{(x+3)}}$$

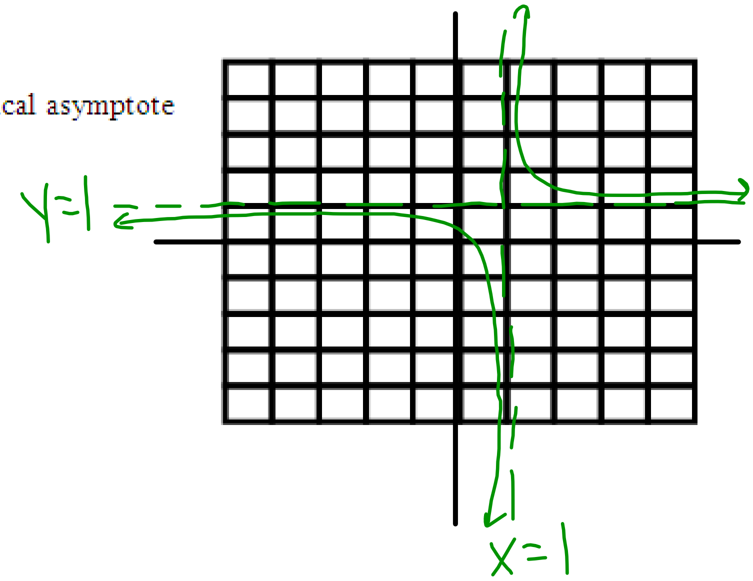
$x=-3$ is a hole

Action

A rational function, $f(x) = p(x)/q(x)$ has a vertical asymptote at $x = a$ if $p(a)/q(a) = p(a)/0$.

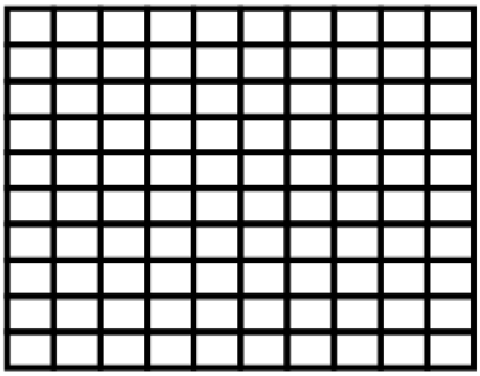
Example: $$\frac{x+2}{x-1}$$

Vertical asymptote at $x=1$



Action

A rational function, f(x) = p(x)/q(x), has a horizontal asymptotes only when the degree of p(x) is less than or equal to the degree of q(x).



Example:

$$\frac{x}{x^2-5} \quad \begin{array}{l} \text{degree} = 1 \\ \text{degree} = 2 \end{array}$$

horizontal asymptote at $y=0$

Action

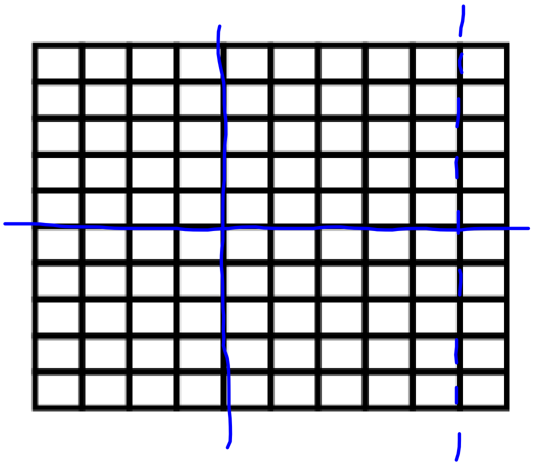
A rational function, f(x) = p(x)/q(x), has an oblique (slant) asymptote only when the degree of p(x) is greater than the degree of q(x) by exactly 1.

Example:

$$\frac{x^2+4x}{x-5}$$

degree = 2

degree = 1



To find the equation of an oblique asymptote, divide the denominator into the numerator. Ignore any remainder.

$$\begin{array}{r|l} & \boxed{x+9} \\ x-5 & x^2+4x \\ & \underline{x^2-5x} \\ & \quad 9x \end{array}$$

Consolidation

Find all asymptotes of the functions below.

1. $f(x) = \frac{x}{3x-6}$

vertical asymptote: set denominator to 0 and solve

$3x - 6 = 0$
$x = 2$

horizontal asymptote: divide the leading coefficients

$y = \frac{1}{3}$

Consolidation

Find all asymptotes of the functions below.

2. $g(x) \overset{0x^2}{=} \frac{x-4}{x^2-5x+6}$

vertical asymptotes: $x^2 - 5x + 6 = 0$

$(x-3)(x-2) = 0$

$x = 2, 3$

horizontal asymptote: $y = \frac{0}{1}$

$y = 0$

Consolidation

Find all asymptotes of the functions below.

3. $h(x) = \frac{3x^2 - 6x}{x^2 - 1}$

vertical asymptotes: $x^2 - 1 = 0$

$(x+1)(x-1) = 0$

$x = -1, 1$

horizontal asymetote: $\frac{3}{1}$

$y = 3$

4. $j(x) = \frac{2x^2-1}{0x^2 x-1}$

vertical asymptote: $x = 1$

horizontal asymptote: $\cancel{y = \frac{2}{0}}$ ~~undefined~~

No horizontal asymptote

oblique asymptote:

$$\begin{array}{r} \boxed{2x+2} \\ x-1 \overline{)\, 2x^2+0x-1} \\ \underline{2x^2-2x} \\ 2x \end{array}$$

Consolidation

page 262 #1-3