Minds On

Solve for x:

$$2\times\frac{(3x+4)}{2} < 5\times 2$$

$3x+4<10$

$3x<6$

$x<2$

$$-2\times\frac{(3x+4)}{-2} < 5\times -2$$

$3x+4>-10$

switch sign (multiplied by negative)

$3x>-14$

$x>-\frac{14}{3}$

Action

Example 1

The function $P(t) = \frac{20t}{t+1}$ models the population, in thousands, of Shelbyville, t years after 1997. The population, in thousands of nearby Springfield is modelled by $Q(t) = \frac{240}{t+8}$. How can you determine the time period when the population of Springfield exceeded the population of Shelbyville?

$$\frac{240}{t+8} > \frac{20t}{t+1}$$

Because we are looking at t years since 1997, t will be positive. This means we can multiply by either (t+8) or (t+1) without worrying about whether the inequality sign will change.

$$(t+1)\frac{240}{t+8} > \frac{20t}{\cancel{t+1}}\cancel{(t+1)}$$

$$\cancel{(t+8)}\frac{240(t+1)}{\cancel{t+8}} > 20t(t+8)$$

$$240(t+1) > 20t(t+8)$$

$$240t+240 > 20t^2+160t$$

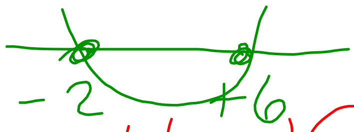
$$-20t^2-160t \qquad -20t^2-160t$$

$$-20t^2+80t+240>0$$

$$\frac{\cancel{-20}(t^2-4t-12)}{-20} > \frac{0}{-20}$$

switch sign

$$t^2-4t-12<0$$



$$t^2 - 4t - 12 < 0$$

$$(t-6)(t+2) < 0$$

We have 3 intervals to look at: $(-\infty, -2)$, $(-2, 6)$, $(6, +\infty)$

	$(-\infty,-2)$ $(t=-3)$	$(-2,6)$ $(t=0)$	$(6,+\infty)$ $(t=7)$
$(t-6)$	$-$	$-$	$+$
$(t+2)$	$-$	$+$	$+$
$P(t)$	$+$	$-$	$+$

Population of Springfield exceeded Shelbyville between 1995 and 2003.

Action

How is the solution to an inequality different from the solution to an equation?

solution is a range of values

a single solution, or a few discrete solutions.

Action

Example 2

Solve $x - 2 < \frac{8}{x}$, x≠0

$x - 2 < \frac{8}{x}$

We can't multiply both sides by x to clear the denominator, because we don't know if x is positive or negative. * we don't know what way the inequality sign would go.

$$x - 2 < \frac{8}{x}$$
$$-\frac{8}{x} \quad -\frac{8}{x}$$

$$\frac{x}{x}(x-2) - \frac{8}{x} < 0$$

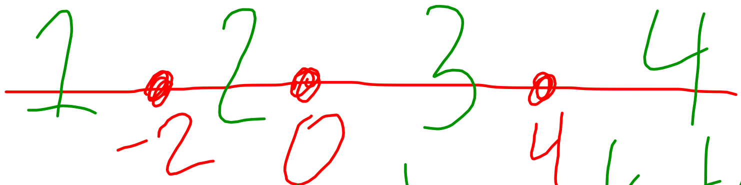
$$\frac{x^2}{x} - \frac{2x}{x} - \frac{8}{x} < 0$$

$$\frac{x^2 - 2x - 8}{x} < 0$$

$$\frac{x^2-2x-8}{x} < 0$$

$$\frac{(x-4)(x+2)}{x} < 0$$

we have vertical asymptote: $x=0$
we have x-intercepts at $x=4,-2$



we have 4 intervals to consider

	$(x=-3)$ $(-\infty,-2)$	$(x=-1)$ $(-2,0)$	$(x=1)$ $(0,4)$	$(x=5)$ $(4,\infty)$
$(x-4)$	$-$	$-$	$-$	$+$
$(x+2)$	$-$	$+$	$+$	$+$
x	$-$	$-$	$+$	$+$
$f(x)$	$-$	$+$	$-$	$+$

$x-2<\frac{8}{x}$ when $(-\infty,-2),(0,4)$

Action

Example 3

Determine the solution set for the inequality $\frac{x+3}{x+1} \geq \frac{x-2}{x-3}$

Again, we don't know the value of x (+ or −) so we can't multiply by $(x+1)$ or $(x-3)$

$$\frac{(x-3)}{(x-3)}\frac{x+3}{x+1} - \frac{x-2}{x-3} \geq 0$$

$$\frac{(x-3)(x+3)}{(x-3)(x+1)} - \frac{x-2}{x-3}\frac{(x+1)}{(x+1)} \geq 0$$

$$\frac{(x-3)(x+3)-(x-2)(x+1)}{(x-3)(x+1)} \geq 0$$

$$\frac{x^2-9-(x^2-x-2)}{(x-3)(x+1)} \geq 0$$

$$\frac{\cancel{x^2}-9\cancel{-x^2}+x+2}{(x-3)(x+1)} \geq 0$$

$$\frac{x-7}{(x-3)(x+1)} \geq 0$$

1 asymptote 2 asymptote 3 x-intercept 4

−1 +3 +7

4 intervals to consider

	$(x=-2)$ $(-\infty,-1)$	$(x=0)$ $(-1,3)$	$(x=4)$ $(3,7)$	$(x=8)$ $(7,+\infty)$
$(x-7)$	$-$	$-$	$-$	$+$
$(x-3)$	$-$	$-$	$+$	$+$
$(x+1)$	$-$	$+$	$+$	$+$
$f(x)$	$-$	$+$	$-$	$+$

because we have $\geq$, you would expect

$[-1,3]$ and $[7,+\infty)$

but it is actually $(-1,3)$ and $[7,+\infty)$

because $x=-1$ and $x=3$ are asymptotes.

Pg. 295

1, 3, 5, 9