

Reminder:

- To find the average rate of change between two points:  
Find the slope of the "secant line" between the two points.

$$foc = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

- To find the instantaneous rate of change:  
Find the slope of the "tangent line" that goes through the point of interest. In our case, we find the slope of a line through our point and another **very nearby** point.

$$foc = \frac{f(a+h) - f(a)}{h}$$

*h is typically 0.001  
a is our value of interest*

**Example:** The revenue equation for toothbrushes is  $R(x) = \frac{5x}{2+x^2}$ , where  $x$  is the number of toothbrushes sold, in thousands, and  $R$  is the ~~price~~, in dollars.

a) Determine the average rate of change for 1200 to 2500 toothbrushes.

$$\text{Ave. Revenue} = \frac{R(2.5) - R(1.2)}{2.5 - 1.2}$$

$$= \frac{\left[ \frac{5(2.5)}{2+(2.5)^2} \right] - \left[ \frac{5(1.2)}{2+(1.2)^2} \right]}{1.3}$$

$$= \frac{2.78 - 1.44}{1.3}$$

$$= 0.69$$

The average revenue is \$0.69/brush.

b) Calculate the instantaneous rate of change for 1500 toothbrushes.

$$R(x) = \frac{5x}{2+x} \quad x = 1.5$$

$$\Delta C = \frac{R(1.5 + 0.001) - R(1.5)}{0.001}$$

$$= \frac{\left[ \frac{5(1.501)}{2+(1.501)} \right] - \left[ \frac{5(1.5)}{2+1.5} \right]}{0.001}$$

$$= \frac{2.1437 - 2.1429}{0.001}$$

$$= 0.8$$

\$0.80 per brush

**Example:** Given the function  $\frac{x}{x+3}$ , determine the instantaneous rate of change of the function when  $x = -3$ .

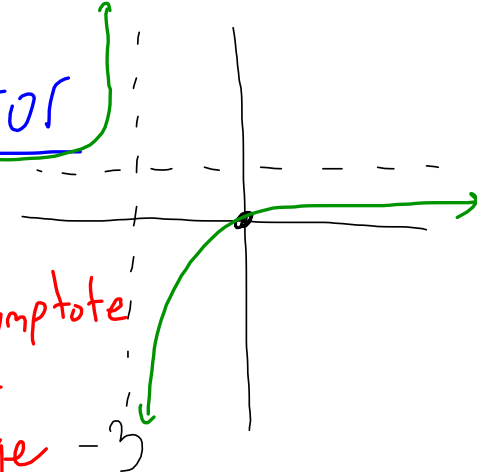
$$\text{slo} = \frac{f(-3+0.001) - f(-3)}{0.001}$$

$$= \frac{f(-2.999) - f(-3)}{0.001}$$

$$= \left[ \frac{-2.999}{-2.999+3} \right] - \left[ \frac{-3}{-3+3} \right]$$

$$\frac{\quad}{0.001}$$

$$f(x) = \frac{x}{x+3}$$

$$= \frac{-2999 - \text{error}}{0.001}$$


Because there is a vertical asymptote at  $x = -3$ , we cannot draw a line tangent to the curve. The rate of change doesn't exist.

**Example:** The snowshoe hare population in a conservation area can be predicted over time by the model  $p(t) = 50 + \frac{2500t^2}{25+t^2}$ , where  $p$  represents the population and  $t$  is the time in years since the area opened. Determine when the hare population will increase most rapidly, in the first 20 years, and estimate the instantaneous rate of change at this time.

when is the graph steepest?

Increasing most rapidly after 2.9 years  
at a rate of 325 hares/year.

# Practice

Pg. 304

6, 7

Review: Page 308

1 - 3, 5 - 12, 15