Solving Equations Involving Exponents

1. Solve. Round your answers to one decimal place.

a. $k^4 = 20$

$\sqrt[4]{k^4} = \sqrt[4]{20}$

$k \doteq 2.1$

b. $750 = 6x^5$

$\frac{750}{6} = \frac{6x^5}{6}$

$\sqrt[5]{125} = \sqrt[5]{x^5}$

$x \doteq 2.6$

c. $200 = \frac{1}{3}\pi r^3$ $\times 3$

$3\times$

$\frac{600}{\pi} = \frac{\pi r^3}{\pi}$

$r^3 = 191.0$

$r = 5.8$

d. $\sqrt[7]{p^7} = \sqrt[7]{-298}$

$p \doteq -2.3$

e. $643 = 4r^2$

$\frac{643}{4} = \frac{4r^2}{4}$

$\sqrt{r^2} = \sqrt{\frac{643}{4}}$

$r \doteq 17.9$

f. $1540 = \frac{4}{3}\pi r^3$ $\times 3$

$3\times$

$\frac{4620}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{r^3} = \sqrt[3]{\frac{4620}{4\pi}}$

$r \doteq 9.0$

2. Solve each equation to one decimal place. Simplify the expression first, if possible, then use systematic trial with a calculator.

a. $2^x = 12$

x is between 3.5 and 3.6

b. $3^n = 50$

x is between 3.5 and 3.6

c. $5000 = 500(1.05)^t$

$\frac{5000}{500} = \frac{500(1.05)^t}{500}$

$1.05^t = 10$

t is between 47 and 48

d. $2^k = 100$

k is between 6.6 and 6.7

e. $4(10)^m = 500{,}000$

$\frac{4(10)^m}{4} = \frac{500{,}000}{4}$

$10^m = 125000$

m is between 5.0 and 5.1

f. $3200 = 40(1.35)^b$

$\frac{3200}{40} = \frac{40(1.35)^b}{40}$

$1.35^b = 80$

$b \doteq 14.6$

3. A ball is dropped and bounces several times, losing some of its rebound height after each bounch. The height reached, h, in metres, after n bounces is given by the equation $h = 1.5(0.75)^n$.
   a. What is the maximum height after
      i) The first bounce?
      ii) The second bounce?
      iii) The fifth bounce?
   b. From what height was the ball initially dropped?
   c. Determine how many bounces it will take before the ball's rebound height is less than 1% of its initial drop height.

a) i) $h = 1.5(0.75)^1$
$= 1.125\,m$

ii) $h = 1.5(0.75)^2$
$h = 0.84\,m$

iii) $h = 1.5(0.75)^5$
$= 0.36\,m$

b) $h = 1.5(0.75)^0$ (= 1)
$h = 1.5 \times 1$
$h = 1.5\,m$

c) 1% of 1.5 = 0.015 m

$\frac{0.015}{1.5} = \frac{1.5(0.75)^n}{1.5}$

$0.01 = 0.75^n$

After 16 bounces

4. The volume, V, of a sphere is related to its radius, r, by the equation $V = \frac{4}{3}\pi r^3$. If it takes 42,400 cm^3 tp inflate six identical balls, what is the radius of each ball?

Volume = 42400 × 6
= 254,400

$3 \times 254{,}400 = \frac{4}{3}\pi r^3 \times 3$

$\frac{763200}{4\pi} = \frac{4\pi r^3}{4\pi}$

$\sqrt[3]{r^3} = \sqrt[3]{60733.5}$

$r = 39.3\,cm$