

Name: _____

Unit 2: Practice Test

1. First, write as a single exponent. Then evaluate. Show all your work for full marks.

$$\begin{aligned} \text{a) } 3^2 \times 3^5 & \\ &= 3^{2+5} \\ &= 3^7 \\ &= 2187 \end{aligned}$$

$$\begin{aligned} \text{b) } (5^{-2})^{-7} & \\ &= 5^{(-2)(-7)} \\ &= 5^{14} \\ &= 6,103,515,625 \end{aligned}$$

$$\begin{aligned} \text{c) } 32^0 & \\ &= 1 \end{aligned}$$

2. Simplify. Write answers with a positive exponent.

$$\begin{aligned} \text{a) } n^{40} \div n^{-2} & \\ &= n^{40--2} \\ &= n^{42} \end{aligned}$$

$$\begin{aligned} \text{b) } (x^2)^{-3} & \\ &= x^{(2)(-3)} \\ &= x^{-6} = \frac{1}{x^6} \end{aligned}$$

$$\begin{aligned} \text{c) } (x^{-2} z y^5)^{-2} & \\ &= x^{(-2)(-2)} z^{-2} y^{(-5)(-2)} \\ &= x^4 z^{-2} y^{10} \\ &= \frac{x^4 y^{10}}{z^2} \end{aligned}$$

$$\begin{aligned} \text{d) } \left(\frac{x^7}{y^8} \right)^{-2} & \\ &= \frac{x^{(7)(-2)}}{y^{(8)(-2)}} = \frac{x^{-14}}{y^{-16}} \\ &= \frac{y^{16}}{x^{14}} \end{aligned}$$

3. Express each of the following powers with a radical (root sign), then evaluate.

$$\begin{aligned} \text{a) } 25^{\frac{1}{2}} & \\ &= \sqrt{25} \\ &= 5 \end{aligned}$$

$$\begin{aligned} \text{b) } 32^{\frac{3}{5}} & \\ &= \sqrt[5]{32^3} \\ &= 8 \end{aligned}$$

$$\begin{aligned} \text{c) } 16^{\frac{-1}{4}} & \\ &= \sqrt[4]{16}^{-1} \\ &= (2)^{-1} \\ &= \frac{1}{2} \end{aligned}$$

$$\begin{aligned} \text{d) } 64^{\frac{-2}{3}} & \\ &= \sqrt[3]{64}^{-2} \\ &= 4^{-2} = \frac{1}{4^2} = \frac{1}{16} \end{aligned}$$

4. Write each power as a power with base 2.

$$\begin{aligned} \text{a) } 16^3 &= (2^4)^3 \\ &= 2^{12} \end{aligned}$$

$$\begin{aligned} \text{b) } 64^2 &= (2^6)^2 \\ &= 2^{12} \end{aligned}$$

$$\begin{aligned} \text{c) } 8^1 &= (2^3)^1 \\ &= 2^3 \end{aligned}$$

5. Determine the value of x that satisfies the following equations.

$$\text{a) } 2^x = 32$$

$$x = 5$$

$$\begin{aligned} \text{b) } 16^{x-1} &= 2^{3x} \\ (2^4)^{x-1} &= 2^{3x} \\ 4(x-1) &= 3x \\ 4x - 4 &= 3x \\ \begin{array}{r} 4x - 4 = 3x \\ -3x \quad -3x \\ \hline x - 4 = 0 \\ +4 \quad +4 \\ \hline x = 4 \end{array} \end{aligned}$$

$$\begin{aligned} \text{c) } 3^{x+5} &= 9^{x+1} \\ 3^{x+5} &= (3^2)^{x+1} \\ x+5 &= 2(x+1) \\ x+5 &= 2x+2 \\ \begin{array}{r} x+5 = 2x+2 \\ -x \quad -x \\ \hline 5 = x+2 \\ -2 \quad -2 \\ \hline x = 3 \end{array} \end{aligned}$$

6. For each of the following questions, the student writing the solution has made one or more errors. Identify the error(s) and describe what the person did wrong. Write the correct solution.

$$\text{a) } 3^4 \times 3^5 = 3^{20}$$

They multiplied the exponents when they should have added them.

$$3^4 \times 3^5 = 3^9$$

$$\text{b) } (-3xy^2)^3 = -3x^3y^5$$

1. They forgot to apply the exponent to the coefficient.
2. They added to the exponent on y when they should have multiplied.

$$(-3xy^2)^3 = -27x^3y^6$$

7. Explain how you would use technology to find a solution to the equation $2^x = 3^{3x+1}$.

You would graph functions as $y=2^x$ and $y=3^{3x+1}$, then you would find their point of intersection. The x-value of that point is the solution.

8. The population of a bacteria colony, p , is doubling every day, d , according to the equation $p = 2^d$. Five days later, the population of another bacteria colony, q , is quadrupling every day, according to the equation $q = 4^{d-5}$.

When will the populations of the two bacteria be equal?

What will each population be?

$$2^d = 4^{d-5}$$

$$2^d = (2^2)^{d-5}$$

$$d = 2(d-5)$$

$$d = 2d - 10$$

$$\begin{array}{r} -2d \\ -2d \end{array} \quad -d = -10$$

$$d = 10$$

After 10 days, the populations will be the same.

$$p = 2^{10} = 1024 \checkmark$$

The population will be 1024.

$$p = 4^{d-5}$$

$$p = 4^{10-5}$$

$$p = 4^5$$

$$= 1024 \checkmark$$

9. The approximate population of Canada, P millions, is represented by the exponential function $P = 27.3(1.016)^n$, where n is the number of years since 1991. In what year will the population first go over 50 million people?

$$\frac{50}{27.3} = \frac{27.3(1.016)^n}{27.3}$$

$$1.83 = 1.016^n$$

Solve by systematic guess

n is between 38 and 39

$$1991 + 38 = 2029$$

In the year 2029,
the population will break
50 million.

10. The volume, V , of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$ where r is the radius. If a cone with volume $36,000 \text{ cm}^3$ has a height that is double the diameter, what are the dimensions of the cone?

$$V = \frac{\pi r^2 h}{3}$$

$$h = 2d \quad *d = 2r$$

$$h = 2(2r)$$

$$h = 4r$$

$$3 \times 36000 = \frac{\pi r^2 (4r)}{3} \times 3$$

$$\frac{108000}{4\pi} = \frac{4\pi r^3}{4\pi}$$

$$\sqrt[3]{r^3} = \sqrt[3]{8594.4}$$

$$r = 20.5$$

\therefore the radius is 20.5 cm
the diameter is 41 cm
the height is 82 cm