

1. First, write as a single exponent. Then evaluate, express final answers as integers or fractions, **no decimals**.

a. $4^5 \times 4^7$

$$= 4^{5+7}$$

$$= 4^{12}$$

b. $(3^4)^{-5}$

$$= 3^{-20}$$

$$= \frac{1}{3^{20}}$$

c. $5^2 \div 5^5$

$$= 5^{2-5}$$

$$= 5^{-3}$$

$$= \frac{1}{5^3}$$

2. \oplus Simplify, using the exponent laws. Write answers with positive exponents.

a. $(x^{-2})(x)(x^7)$

$$= x^{-2+1+7}$$

$$= x^6$$

b. $(m^2n^5)(m^{-8}n^{-6})$

$$= m^{2-8}n^{5-6}$$

$$= m^{-6}n^{-1}$$

$$= \frac{1}{m^6n}$$

c. $(y^{-4} \div y^{-8})$

$$= y^{-4-(-8)}$$

$$= y^4$$

d. $(a^{-1}b) \div (a^{-7}b^{-2})$

$$= a^{-1-(-7)}b^{1-(-2)}$$

$$= a^6b^3$$

e. $(2u^{-9})^{-1}$

$$= 2^{-1}u^{-9 \times -1}$$

$$= 2^{-1}u^9$$

$$= \frac{u^9}{2}$$

f. $(4w^3)^{-4}$

$$= 4^{-4}w^{3 \times -4}$$

$$= 4^{-4}w^{-12}$$

$$= \frac{1}{4^4w^{12}}$$

$$= \frac{1}{256w^{12}}$$

3. Solve. Round your answers to one decimal place.

a. $k^5 = 20$

$$k = \sqrt[5]{20}$$

$$k = 1.8$$

b. $\frac{1200}{10} = \frac{10x^5}{10}$

$$x^5 = 120$$

$$x = \sqrt[5]{120}$$

$$x = 2.6$$

c. $100 = \frac{1}{3}\pi r^3$

$$3 \times 100 = \frac{\pi r^3}{3} \times 3$$

$$\frac{\pi r^3}{\pi} = \frac{300}{\pi}$$

$$r^3 = 95.5$$

$$r = \sqrt[3]{95.5}$$

$$r = 4.6$$

4. Express each of the following powers as a radical (root sign), then evaluate. Express final answers as integers or fractions, **no decimals**. (8 marks)

a. $81^{\frac{1}{4}}$

$$= \sqrt[4]{81}$$
$$= 3$$

b. $27^{\frac{-2}{3}}$

$$= (\sqrt[3]{27})^{-2}$$
$$= 3^{-2}$$
$$= \frac{1}{3^2}$$
$$= \frac{1}{9}$$

c. $(-32)^{\frac{3}{5}}$

$$= (\sqrt[5]{-32})^3$$
$$= (-2)^3$$
$$= -8$$

5. \oplus Write each power as a power with base 4 with a single exponent.

a. 64^2

$$= (4^3)^2$$
$$= 4^6$$

b. 2^6

$$= (4^{\frac{1}{2}})^6$$
$$= 4^3$$

c. 21^0

$$= 4^0$$

d. 16^{3x}

$$= (4^2)^{3x}$$
$$= 4^{6x}$$

6. Determine the value of x that satisfies the following equations.

a. $16^{x-2} = 2^{5x}$

$$(2^4)^{x-2} = 2^{5x}$$

$$4(x-2) = 5x$$

$$4x - 8 = 5x$$

$$x = -8$$

b. $3^{4x+2} = 9^{x-1}$

$$3^{4x+2} = (3^2)^{(x-1)}$$

$$4x+2 = 2(x-1)$$

$$4x+2 = \cancel{2x} - 2$$

$$-2x$$

$$2x+2 = -2$$

$$2x = -4$$

$$x = -2$$

c. $4^{3(x-2)} = 64^{2x+5}$

$$4^{3(x-2)} = (4^3)^{2x+5}$$

$$3(x-2) = 3(2x+5)$$

$$3x - 6 = \cancel{6x} + 15$$

$$-6x$$

$$-6x$$

$$-3x - 6 = 15$$

$$+6$$

$$+6$$

$$-3x = 21$$

$$x = -7$$

d. $27^{2x-3} = 9^{2(x-5)}$

$$(3^3)^{2x-3} = (3^2)^{2(x-5)}$$

$$3(2x-3) = 4(x-5)$$

$$6x - 9 = \cancel{4x} - 20$$

$$-4x$$

$$+4x$$

$$2x - 9 = -20$$

$$+9$$

$$+9$$

$$2x = -11$$

$$x = -5.5$$

7. \oplus Solve. Round your answers to one decimal place.

a. $2^x = 64$

$$2^5 = 32$$
$$2^6 = 64$$

$$x = 6$$

b. $2^x = 15$

$$2^3 = 8$$
$$2^4 = 16$$

$$2^{3.5} = 11.3$$

$$2^{3.8} = 13.9$$

$$2^{3.9} = 14.9$$

$$x \doteq 3.9$$

c. $\frac{4000}{800} = \frac{800(1.06)^t}{800}$

$$1.06^t = 5$$

$$1.06^{20} = 3.2$$

$$1.06^{30} = 5.7$$

$$1.06^{28} = 5.1$$

$$1.06^{27.5} = 4.96$$

$$1.06^{27.6} = 4.99$$

$$t \doteq 27.6$$

8. The approximate population of Gravenhurst, P thousands, is represented by the exponential function $P = 10.5(1.023)^n$, where n is the number of years since 2014. If this trend continues, in what year will the population break 30 thousand people?

$$\frac{30}{10.5} = \frac{10.5(1.023)^n}{10.5}$$

$$1.023^n = 2.86$$

guess & check to find n

$$1.023^{40} = 2.48$$

$$1.023^{46} = 2.85$$

$$1.023^{(46.2)} = 2.86$$

46 years from now . . .

2060

9. The volume, V , of a cone is given by the formula $V = \frac{1}{3}\pi r^2 h$ where r is the radius. If a cone with volume 600 m^3 has a radius that is $1/4$ the height. What is the diameter of the cone?

$$r = \frac{h}{4} \quad \underline{\underline{\text{or}}} \quad h = 4r \quad \text{☺}$$

$$V = \frac{\pi r^2 h}{3}$$

$$3 \times 600 = \frac{\pi r^2 (4r)}{3} \times 3$$

$$\frac{1800}{\pi \times 4} = \frac{\pi \times 4 \times r^3}{\pi \times 4}$$

$$\sqrt[3]{r^3} = \sqrt[3]{43.2}$$

$$r = 5.2$$

$$\text{diameter} = 10.4 \text{ m}$$