<u>9.3 Intersection of Two Planes</u>

Minds On: Consider the solution to the system of equations $x - y + z = 4$ and $x - y + z = 5$. Discuss how these planes might be related to each other.

As with the intersection of lines, there are 3 cases for the intersection of two planes:

Case 1: Two planes intersect along a line

Case 2: Two parallel planes

Case 3: Two coincident planes

Solutions for a system of equations representing two planes

The system of equations corresponding to the intersection of two planes will have either zero solutions (parallel) or an infinite number of solutions (a line or planes are coincident). It is not possible for two planes to intersect at a single point.

Example 1: Determine the solution to the following system of equations:

$x + 2y - 3z = -1$ $\qquad$ $4x + 8y - 12z = -4$

Intersection of Two Planes and their Normals

If the planes π_1 and π_2 have $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$ as their respective normals, we know the following:

1. If $\overrightarrow{n_1} = k\overrightarrow{n_2}$ for some scalar, k, the planes are coincident or they are parallel and non-coincident. If they are coincident, there are an infinite number of points of intersection. If they are parallel and non-coincident, there are no points of intersection.

2. If $\overrightarrow{n_1} \neq k\overrightarrow{n_2}$, the two planes intersect in a line. This also results in an infinite number of points of intersection.

Example 2: Determine the solution to the following system of equations:

$x - y + z = 3$ $2x + 2y - 2z = 3$

Example 3: Determine the solution to the following system of equations:

$2x - y + 3z = -2$ $x - 3z = 1$

Example 4: Determine an equation of a line that passes through the point P(5, -2, 3) and is parallel to the line of intersection of the planes π_1: $x + 2y - z = 6$ and π_2: $y + 2z = 1$.