Learning Goal: I will be able to determine the shortest distance from a point to a line.

Minds On: Shortest Distance?

Action: Distance Formulae

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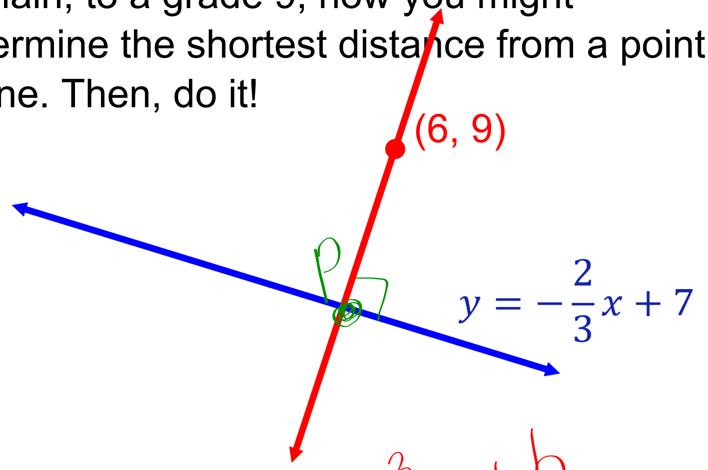
Minds On

Shortest Distance?

Explain, to a grade 9, how you might determine the shortest distance from a point to line. Then, do it!



$y_2 = \frac{3}{2}x + b$

<u>Find b</u>

$b = y - \frac{3}{2}x$

$b = (9) - \frac{3}{2}(6)$

$b = 9 - \frac{18}{2}$

$b = 0$

$\therefore y_2 = \frac{3}{2}x$

$$y_1 = -\frac{2}{3}x + 7$$

$$y_2 = \frac{3}{2}x$$

Find P

$$-\frac{2}{3}x + 7 = \frac{3}{2}x$$

$$-\frac{2}{3}x - \frac{3}{2}x = -7$$

$$\frac{4}{6}x + \frac{9}{6}x = 7$$

$$\frac{13}{6}x = 7$$

$$x = \boxed{\frac{42}{13}}$$

$$y = \frac{3}{2}\left(\frac{42}{13}\right)$$

$$y = \frac{126}{26} = \boxed{\frac{63}{13}}$$

Distance btw $(6, 9)$ and $\left(\frac{42}{13}, \frac{63}{13}\right)$

$$d = \sqrt{\left(\frac{42}{13} - 6\right)^2 + \left(\frac{63}{13} - 9\right)^2}$$

$$d = \sqrt{\left(\frac{42-78}{13}\right)^2 + \left(\frac{63-117}{13}\right)^2}$$

$$d = \sqrt{\left(\frac{-36}{13}\right)^2 + \left(\frac{-54}{13}\right)^2}$$

$$d = \sqrt{\frac{1296}{169} + \frac{2916}{169}}$$

$$d = \sqrt{\frac{4212}{169}}$$

$$d = \sqrt{24.923}$$

$$d \doteq 4.99$$

9.4 The Distance from a Point to a Line in R^2 and R^3

Distance from a Point $P_0(x_0, y_0)$ to the Line with Equation $Ax + By + C = 0$ in R^2.

known

$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$, where d represents the distance between point $P_0(x_0, y_0)$, and the line defined by $Ax + By + C = 0$, where the point does not lie on the line. But we don't really need this since the formula gives the correct value of 0 when the point does lie on the line.

Example 1: Determine the distance from point P(-6, 4) to the line 5x – 3y + 15 = 0.

x_0 y_0

A B C

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|(5)(-6) + (-3)(4) + (15)|}{\sqrt{(5)^2 + (-3)^2}}$$

$$d = \frac{|(-30) + (-12) + (15)|}{\sqrt{25 + 9}}$$

$$d = \frac{|-27|}{\sqrt{34}}$$

$$d = \frac{27}{\sqrt{34}}$$

$$d \doteq 4.63 \text{ units}$$

Example 2: Determine the distance from point P(15, -9) to the line $\vec{r}$ = (-2, -1) + s(-4, 3), s ∈ R.

need in

$Ax+By+C=0$

$\vec{r}$ Parametric

$x=-2-4s$

$y=-1+3s$

$\vec{r}$ symmetric

$x=-2-4s$

$4s=-2-x$

$s=\frac{-2-x}{4}$

$y=-1+3s$

$3s=y+1$

$s=\frac{y+1}{3}$

$$\frac{-2-x}{4}=\frac{y+1}{3}$$

$$3(-2-x)=4(y+1)$$

$$-6-3x=4y+4$$

$$\boxed{3x+4y+10=0}$$

OR

$\vec{m} = (-4, 3)$

$\therefore \vec{n} = (3, 4)$

A   B

$Ax + By + C = 0$

$3x + 4y + C = 0$

$(x, y) = (-2, -1)$

$3(-2) + 4(-1) + C = 0$

$-10 + C = 0$

$C = 10$

$\therefore \boxed{3x + 4y + 10 = 0}$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|(3)(15) + (4)(-9) + 10|}{\sqrt{(3)^2 + (4)^2}}$$

$$d = \frac{|45 - 36 + 10|}{\sqrt{25}}$$

$$d = \frac{19}{5} = 3.8$$

Example 3: Calculate the distance between the two parallel lines $5x - 12y + 60 = 0$ and $5x - 12y - 60 = 0$

Need a point!

Choose "any" x or y-value and find the other value on one of the lines.

Let $\boxed{x=0}$ on L_1

$5(0) - 12y + 60 = 0$

$\boxed{y = 5}$

Now, use the formula with the point we just found and L_2.

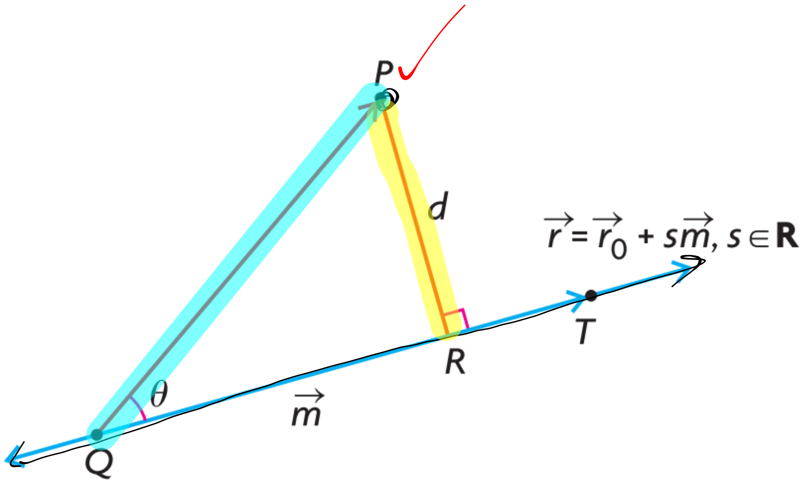
$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$d = \frac{|(5)(0) - 12(5) - 60|}{\sqrt{5^2 + (-12)^2}}$$

$$d = \frac{|-120|}{13}$$

$$d = \frac{120}{13} \doteq 9.23$$

Distance from a Point to a Line in R^3



1. Define *d* in terms of QP and θ.

$$\sin\theta = \frac{d}{|QP|}$$

$$\boxed{d = |QP|\sin\theta}$$

2. Introduce the cross product formula with m and QP.

$$|\vec{m} \times \vec{QP}| = |\vec{m}||\vec{QP}|\sin\theta$$

3. Substitute and rearrange to solve for *d*.

$$|\vec{m} \times \vec{QP}| = |\vec{m}|\, d$$

$$d = \frac{|\vec{m} \times \vec{QP}|}{|\vec{m}|}$$

where $\vec{m}$ is direction vector of given line

P is the point given

Q is any point on the given line

Distance, d, from a Point, P, to the line $\vec{r} = \vec{r_0} + s\vec{m}, s \in R.$

In R^3, $d = \frac{|\vec{m} \times \overrightarrow{QP}|}{|\vec{m}|}$, where Q is a point on the line and P is any other point, both of whose coordinates are known, and $\vec{m}$ is the direction vector of the line.

Example 4: Determine the distance from point P(-1, 1, 6) to the line with equation $\vec{r} = (1, 2, -1) + t(0, 1, 1), t \in R$.

$$d = \frac{|\vec{m} \times \vec{QP}|}{|\vec{m}|}$$

$\vec{m} = (0, 1, 1)$

$P = (-1, 1, 6)$

$Q = (1, 2, -1)$

$\vec{QP} = ((-1)-(1), (1)-(2), (6)-(-1))$

$$\boxed{\vec{QP} = (-2, -1, 7)}$$

$$d = \frac{|(0,1,1) \times (-2,-1,7)|}{\sqrt{(0)^2+(1)^2+(1)^2}}$$

$\vec{m} \times \vec{QP}$

$$\begin{array}{cc} 1 & -1 \\ 1 & 7 \\ 0 & -2 \\ 1 & -1 \end{array}$$

$x = 8$

$y = -2$

$z = 2$

$$d = \frac{|(8, -2, 2)|}{\sqrt{2}}$$

$$d = \frac{\sqrt{64+4+4}}{\sqrt{2}}$$

$$d = \sqrt{\frac{72}{2}}$$

$$d = \sqrt{36}$$

$$d = 6$$

Consolidation

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