I will be able to solve systems of equations using matrices.

Solve the following system

Do this!

$$3x - 5y = 10 \quad (1)$$

$$x + 3y = 8 \quad (2)$$

a) By substitution

From (2)

$x = -3y + 8 \quad (3)$

Sub (3) into (1)

$3(-3y+8) - 5y = 10$

$-9y + 24 - 5y = 10$

$\boxed{y = 1}$

sub $y = 1$ into (2)

$x + 3(1) = 8$

$\boxed{x = 5}$

b) By elimination

$$
\begin{array}{rl}
 & 3x - 5y = 10 \quad (1) \\
+ & -3x - 9y = -24 \quad (2)\times -3 \\
\hline
 & -14y = -14
\end{array}
$$

$\boxed{y = 1}$

sub $y = 1$ into (2)

$x + 3(1) = 8$

$\boxed{x = 5}$

c) Using matrices

$$\begin{array}{cc} x & y \end{array}$$

$$\left[\begin{array}{cc|c} 3 & -5 & 10 \\ 1 & 3 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 3 & 8 \\ 3 & -5 & 10 \end{array}\right] \quad ① \leftrightarrow ②$$

$$\left[\begin{array}{cc|c} 1 & 3 & 8 \\ 0 & -14 & -14 \end{array}\right] \quad -3① + ②$$

Gaussian

$$\frac{-14y = -14}{\boxed{y = 1}}$$

sub $y = 1$ into ①

$$x + 3(1) = 8$$

$$\boxed{x = 5}$$

c) Using matrices

Gauss-Jordan Elimination

$$\begin{array}{cc} x & y \\ \end{array}$$
$$\left[\begin{array}{cc|c} 3 & -5 & 10 \\ 1 & 3 & 8 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & 3 & 8 \\ 3 & -5 & 10 \end{array}\right] \quad ① \leftrightarrow ②$$

$$\left[\begin{array}{cc|c} 1 & 3 & 8 \\ 0 & -14 & -14 \end{array}\right] \quad -3① + ②$$

$$\left[\begin{array}{cc|c} 1 & 3 & 8 \\ 0 & 1 & 1 \end{array}\right] \quad -\frac{1}{14}②$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 1 \end{array}\right] \quad -3② + ①$$

$$\boxed{\therefore x = 5, y = 1}$$

From Equations to a Matrix

$$2x - 3y + 4z = 5$$

$$x - y + 3z = 7$$

$$3x + y - z = -4$$

Coefficient Matrix

$$\begin{bmatrix} 2 & -3 & 4 \\ 1 & -1 & 3 \\ 3 & 1 & -1 \end{bmatrix}$$

Augmented Matrix

$$\left[\begin{array}{ccc|c} 2 & -3 & 4 & 5 \\ 1 & -1 & 3 & 7 \\ 3 & 1 & -1 & -4 \end{array}\right]$$

Legal Operations

1. Switch rows

    - Always look for a coeffient of 1 on x-term
    then switch to top

2. Multiply a row by a nonzero constant.

3. Add a multiple of one row to another.

***row = equation**

Gaussian Elimination

(row-echelon form)

Solve using elementary row operations.

$$2x + 3y - z = 1$$
$$x + 2y + 2z = 9$$
$$x + y = 1$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & 1 \\ 1 & 2 & 2 & 9 \\ 1 & 1 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 1 & 2 & 2 & 9 \\ 2 & 3 & -1 & 1 \end{array}\right] \quad (1) \leftrightarrow (3)$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 8 \\ 0 & 1 & -1 & -1 \end{array}\right] \quad \begin{array}{l} -1(1)+(2) \\ -2(1)+(3) \end{array}$$

✓ gaussian

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & -3 & -9 \end{array}\right] \quad -1(2)+(3)$$

diagonal of 1s

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad \frac{1}{3}(3)$$

make 0s!

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad -2(3)+(2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right] \quad -1(2)+(1)$$

$\therefore (x, y, z) = (-1, 2, 3)$

* check in original equations

Gauss- Jordan Elimination

(reduced row-echelon form)

$$\begin{aligned} x - y + 2z &= -9 \\ -x - y + 2z &= -7 \\ x + 2y - z &= 6 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -9 \\ -1 & -1 & 2 & -7 \\ 1 & 2 & -1 & 6 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -9 \\ 0 & -2 & 4 & -16 \\ 0 & 3 & -3 & 15 \end{array}\right] \begin{array}{l} \\ (1)+(2) \\ -1(1)+(3) \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -9 \\ 0 & 1 & -2 & 8 \\ 0 & 3 & -3 & 15 \end{array}\right] -\tfrac{1}{2}②$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -9 \\ 0 & 1 & -2 & 8 \\ 0 & 0 & 3 & -9 \end{array}\right] -3② + ③$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -9 \\ 0 & 1 & -2 & 8 \\ 0 & 0 & 1 & -3 \end{array}\right] \tfrac{1}{3}③$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & -9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad 2(3)+(2)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & -7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad (2)+(1)$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{array}\right] \quad -2(3)+(1)$$

$\therefore (x, y, z) = (-1, 2, -3)$

I may be a few minutes late.

On the following 3 slides is my solution to the assigned question from yesterday (3b)

Please go through the slides slowly so everyone can check their work.

Once everyone has got their solutions verified, move on to the next slide (one more matrix algebra question).

I have provided a solution on the slides that follow. After that, move to the last slide.

Pg 595 3b

$$3x - 2y + z = 6$$
$$x - 3y - 2z = -26$$
$$-x + y + z = 9$$

$$\left[\begin{array}{ccc|c} 3 & -2 & 1 & 6 \\ 1 & -3 & -2 & -26 \\ -1 & 1 & 1 & 9 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & -26 \\ 3 & -2 & 1 & 6 \\ -1 & 1 & 1 & 9 \end{array}\right] \quad 1 \longleftrightarrow 2$$

$$\begin{array}{cl} \left[\begin{array}{cccc} 1 & -3 & -2 & -26 \\ 0 & 7 & 7 & 84 \\ 0 & -2 & -1 & -17 \end{array}\right] & \begin{array}{l} \\ -3(1)+(2) \\ (1)+(3) \end{array} \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & -26 \\ 0 & 1 & 1 & 12 \\ 0 & -2 & -1 & -17 \end{array}\right] \frac{1}{7}②$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & -26 \\ 0 & 1 & 1 & 12 \\ 0 & 0 & 1 & 7 \end{array}\right] 2② + ③$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & -26 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \end{array}\right] -③ + ②$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 0 & -12 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \end{array}\right] 2③ + ①$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 7 \end{array}\right] \quad 3(2) + (1)$$

$\therefore (x, y, z) = (3, 5, 7)$

Solve the system using Gauss-Jordan elimination.

$$2x + y + 2z = 2$$
$$x + y + z = 2$$
$$x + z = 0$$

$$\left[\begin{array}{ccc|c} 2 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 \\ 1 & 0 & 1 & 0 \end{array}\right]$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 2 & 1 & 2 & 2 \end{bmatrix} \quad 1 \longleftrightarrow 3$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 1 & 0 & 2 \end{bmatrix} \begin{matrix} \\ -1+2 \\ -2(1)+3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} \\ \\ -2+3 \end{matrix}$$

row of zeroes.. 2 equations w/ 3 unknowns... infinite solutions

Obviously $y=2$, let $z=t$ and $x+t=0$, $x=-t$

$(x,y,z)=(-t,2,t)$

Intersection is a line

$\vec{r}=(0,2,0)+t(-1,0,1)$

That's it for the course!

Use the rest of the period to

a) Review for tomorrow's test

b) Work on culminating

- in red bin on bottom shelf