Learning Goal: I will be able to identify increasing and decreasing parts of functions and use this information to connect the graphs of functions and their derivatives.

Minds On: Whiteboards - increasing or decreasing and what it means

Action:
1. Class note + practice
2. Group task

Consolidation: Working Backwards

Minds On

On your whiteboard, draw a function of your choice.

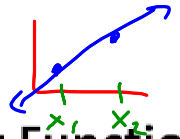
Find and label an interval of your graph that is increasing.

Find and label an interval of your graph that is decreasing.

What can you say about the tangents to the function
for each of these intervals?

How does this relate to derivatives?

Action



4.1 Increasing and Decreasing Functions

A function is increasing over an interval if the slopes of the tangents over that interval are positive. Symbolically: If $x_1 < x_2$, then $f(x_1) < f(x_2)$.

A function is decreasing over an interval is the slopes of the tangents over that interval are negative. Symbolically: If $x_1 < x_2$, then $f(x_1) > f(x_2)$.

For a continuous & differentiable function, f, the function values (y-values) are increasing for all x-values where $\boxed{f'(x) > 0}$, and the function values are decreasing for all x-values where $\boxed{f'(x) < 0}$.

Action

Example 1: Graph the following functions. Use the graph to estimate the values of x for which the function values are increasing, and the values of x for which the y-values are decreasing. Verify your estimates with an algebraic solution.

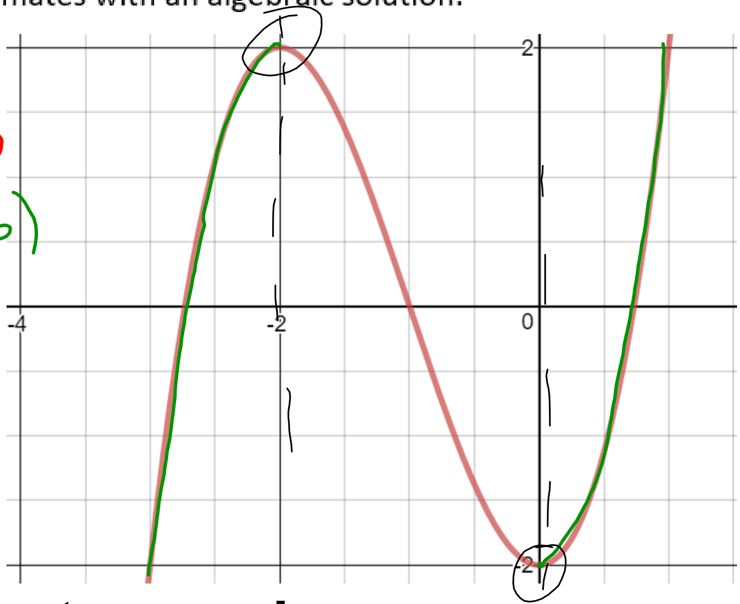
a) $y = x^3 + 3x^2 - 2$

increasing: $x < -2$, $x > 0$

$(-\infty, -2), (0, +\infty)$

decreasing: $-2 < x < 0$

$(-2, 0)$



Without a graph

Find when the derivative is 0 (graph levels out)

Test intervals on either side of these points to find increasing and decreasing intervals. If derivative is positive, increasing, if derivative is negative, decreasing.

$y = x^3 + 3x^2 - 2$

$y' = 3x^2 + 6x$

$y' = 0$ when $3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0, x = -2$

Test

	$x < -2$	$-2 < x < 0$	$x > 0$
$3x$	$-$	$-$	$+$
$x+2$	$-$	$+$	$+$
y'	$+$	$-$	$+$
$\uparrow$ or $\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$

$\therefore$ y is increasing when $x < -2$ and when $x > 0$,

y is decreasing when $-2 < x < 0$.

Action

b) $y = \frac{x}{x^2+1}$

$$y = x(x^2+1)^{-1}$$

$$y' = (1)(x^2+1)^{-1} + (x)(-1)(x^2+1)^{-2}(2x)$$

$$y' = \frac{1}{x^2+1}\cdot\frac{(x^2+1)}{(x^2+1)} - \frac{2x^2}{(x^2+1)^2}$$

$$y' = \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$y' = \frac{-1x^2+1}{(x^2+1)^2}$$

To find peaks/valleys set $y' = 0$

$$\frac{-1x^2+1}{\cancel{(x^2+1)^2}} = 0$$

$$-1x^2+1 = 0$$

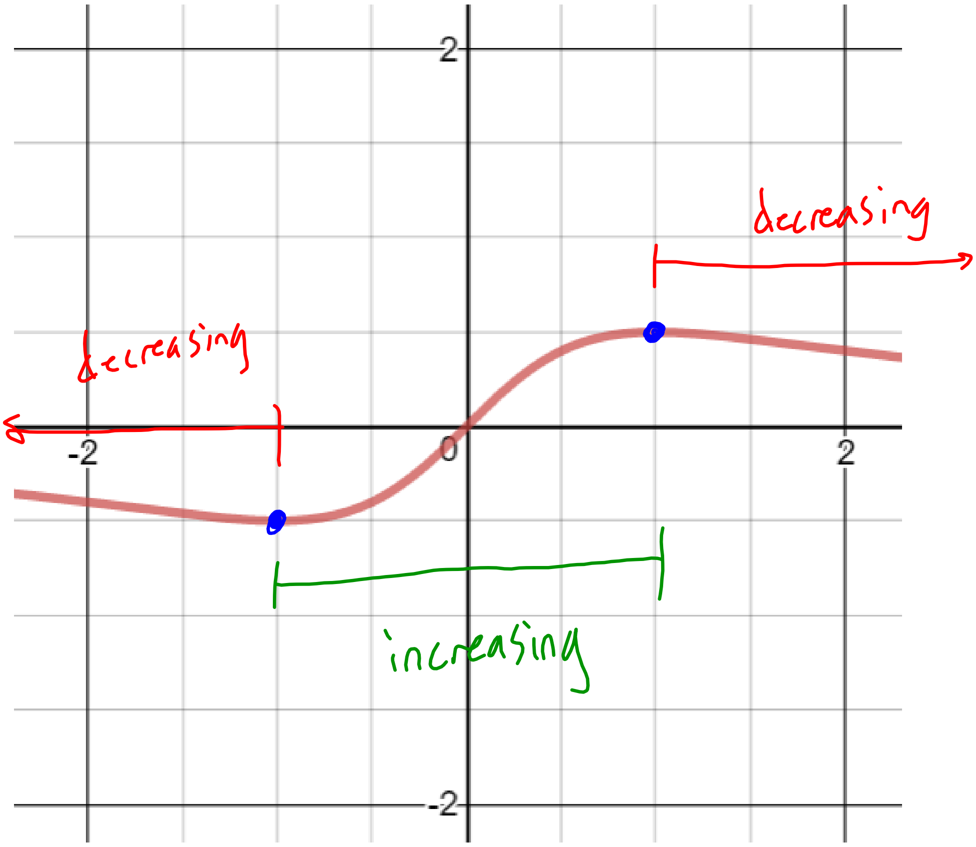
$$x^2 = 1$$

$$x = \pm 1$$

look at intervals

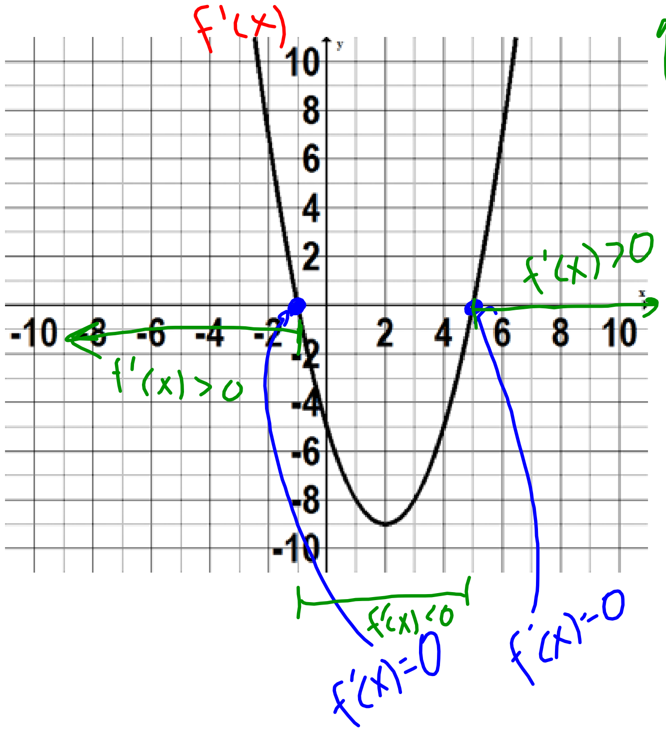
	$x < -1$	$-1 < x < 1$	$x > 1$
$-1x^2+1$	$-$	$+$	$-$
$(x^2+1)^2$	$+$	$+$	$+$
y'	$-$	$+$	$-$

y is increasing when $-1 < x < 1$ and decreasing when $x < -1$ and when $x > 1$.

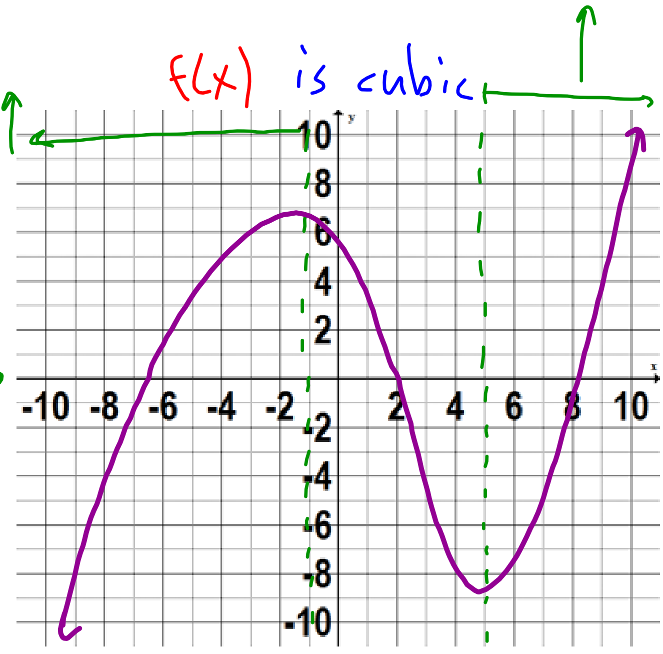

2
-2
0
2
-2
decreasing
decreasing
increasing

Action

Example 2: Consider the graph of f'(x). Graph f(x).



f(x) is cubic



when x = -1 and 5, f(x) has a turning point.

Consolidation

Working Backwards

The equation of f'(x) from the previous example was f'(x) = (x + 1)(x - 5).

Determine the equation of the original function, f(x).

$f'(x) = 1x^2 - 4x - 5$

We worked backwards to figure out what the values would have been BEFORE we took the derivative. The last term can be anything, because when you take the derivative of a constant, it is 0 (it disappears).

$f(x) = \frac{1}{3}x^3 - \frac{}{2}x^2 - 5x + C$