Learning Goal: I will be able to identify critical numbers and determine whether they are a max, a a min or neither.

Minds On: Whiteboards - critical points

Action: Class note + practice

Consolidation: Exit Question

A function f(x) is increasing if: $f'(x) > 0$

A function f(x) is decreasing if: $f'(x) < 0$

Minds On

Draw a function of your choice that has at least one local maximum and one local minimum.

slope is (+)

Draw a tangent to the curve on the left of your local max. Draw another tangent to the curve on the right of your local max.

slope is (−)

What do you notice about the slopes of these tangents?

Does this hold true for all local maximums? Yes

Repeat the process for your local min.

left of min: decreasing
right of min: increasing

Action

Important

4.2 Critical Points, Local Maxima, and Local Minima

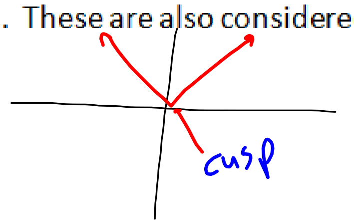
A **critical number** is a number, c, in the domain of f(x) such that $\boxed{f'(x) = 0}$ or $\boxed{f'(x) \text{ is undefined}}$. As a result, (c, f(c)) is called a critical point and usually corresponds to local or absolute extrema.

The First Derivative Test

Let c be a critical number of a function f. When moving through x-values from left to right:

- If f'(x) changes from negative to positive at c, then (c, f(c)) is a local minimum of f.
- If f'(x) changes from positive to negative at c, then (c, ,f(c)) is a local maximum of f.
- If f'(x) does not change its sign at c, then (c, f(c)) is neither a local min nor a local max.

Remember from Chapter #2 that derivatives that don't exist at cusps and corners on a function's graph. These are also considered extrema, and f'(c) = 0 at these points.



Action

when $y'=0$

Example 1: For the function $y = x^4 - 8x^3 + 18x^2$, determine all the critical numbers. Determine whether each of these values of x gives a local max, a local min, or neither for the function.

$y' = 4x^3 - 24x^2 + 36x$

when $y'=0$, $4x^3 - 24x^2 + 36x = 0$

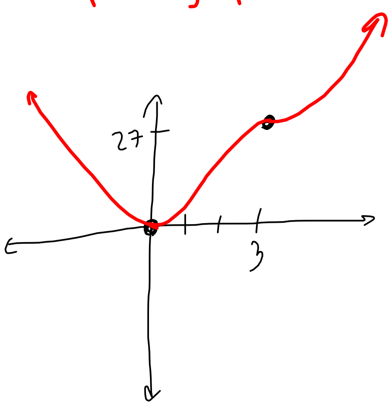
$4x(x^2 - 6x + 9) = 0$

$4x(x-3)^2 = 0$

critical numbers: $x = 0, 3$

critical points: $(0,0)$ and $(3,27)$

Interval	$x<0$	$0<x<3$	$x>3$
$4x$	−	+	+
$(x-3)^2$	+	+	+
y'	−	+	+
slope of graph	↘	↗	↗



Local min at $(0,0)$

Neither max nor min at $(3,27)$

Action

Example 2: Determine whether or not the function $f(x) = x^3$ has a max or min at $(c, f(c))$, where $f'(c) = 0$.

$f'(x) = 3x^2$

when $f'(x) = 0$, $3x^2 = 0$

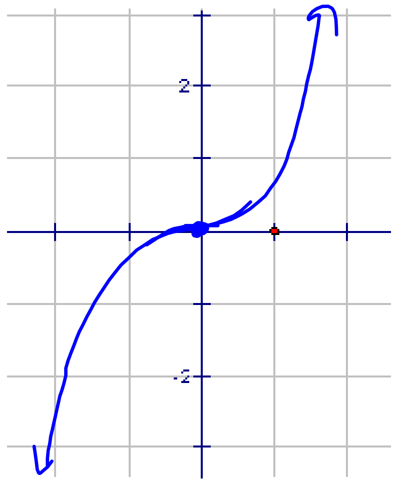
$x^2 = 0$

$x = \pm 0$

$x = 0$

critical number: $x = 0$

critical point: $(0,0)$



$3x^2$ is never < 0, so $(0,0)$ is not a max nor a min.

function is always icreasing ($f'(x)$ is always positive)

Action

Example 3: For the function $f(x) = (x + 2)^{2/3}$, determine the critical numbers. Use your calculator to sketch a graph of the function.

$\sqrt[3]{x+2}^{\,2}$ * f(x) always (+)

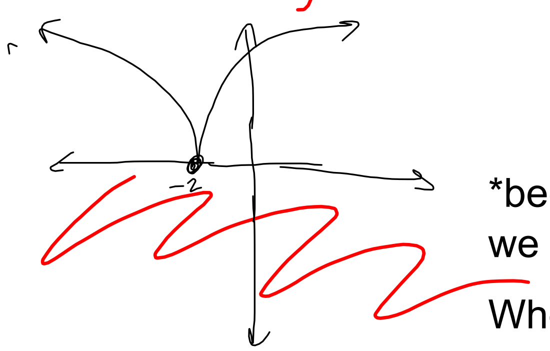
$$f'(x) = \frac{2}{3}(x+2)^{-\frac{1}{3}}(1)$$
$$= \frac{2}{3(x+2)^{1/3}}$$
or
$$= \frac{2}{3\sqrt[3]{x+2}}$$

when $f'(x) = 0$,
$$\frac{2}{3\sqrt[3]{x+2}} = 0$$
$2 = 0$ $\therefore f'(x) \neq 0$ ever

$f'(x)$ is undefined when $x = -2$

$x \to -2^-$	$f'(x)$	$x \to -2^+$	$f'(x)$
-2.1	-1.44	-1.9 EAR	1.44
-2.01	-3.09	-1.99 JJ	3.09
-2.001	-6.67	-1.999 GPM	6.67
-2.0001	-14.36	-1.9999 CVS	14.36

when $x = -2$, $f(x) = 0$ (it is defined, so we don't have an asymptote!)
Must be a cusp.



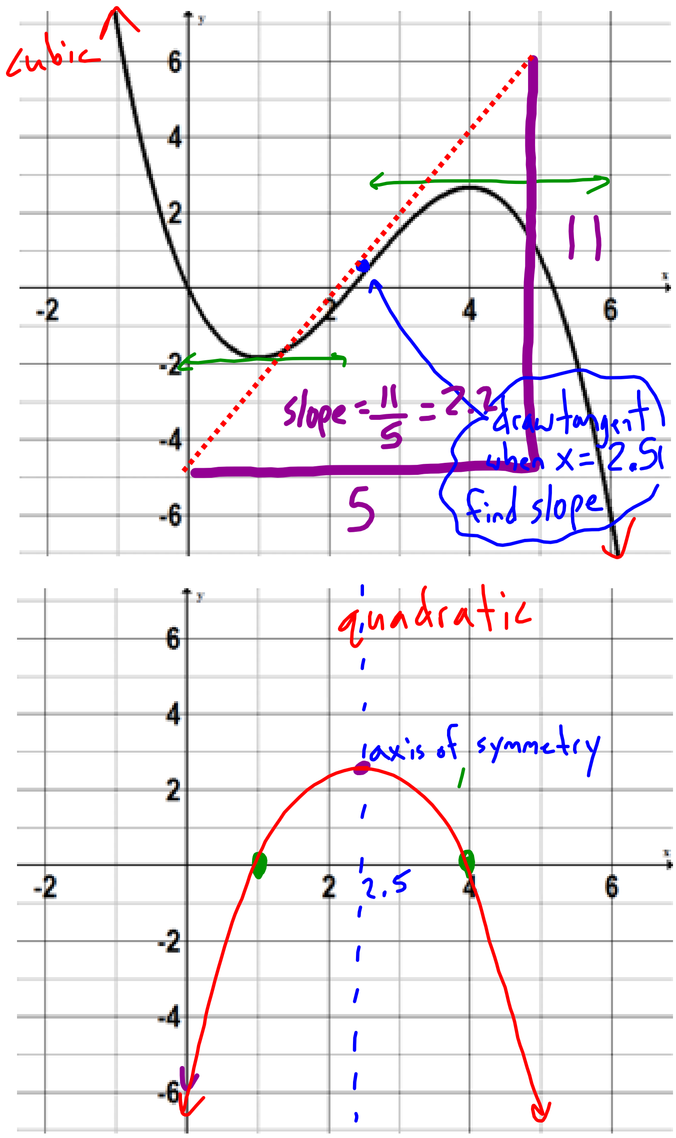
*because f'(x) is undefined when x = -2, we have a critical number x = -2.

When x = -2, f(x) = 0.

Therefore our critical point is (-2,0)

Action

Example 4: Given the graph of a polynomial function y = f(x), graph y = f′(x).



$f'(x) < 0$ when $x < 1$ and when $x > 4$

$f'(x) > 0$ when $1 < x < 4$

Consolidation

Determine the nature of all critical points of f(x) given below, then sketch a rough graph of the function.

$$f(x) = -3x^4 + 8x^3$$