This unit is difficult.

Practice sooner than later.

Minds On

Definition of the Derivative

Together we will determine the derivative of

f(x) = 2^x^

using the definition of the derivative.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

if $f(x) = 2^x$

$$f'(x) = \lim_{h \to 0} \frac{2^{x+h} - 2^x}{h}$$

$$= \lim_{h \to 0} \frac{2^x(2^h - 1)}{h}$$

$$= 2^x \cdot \lim_{h \to 0} \frac{2^h - 1}{h}$$

let's set $h = 0.001$

$$= 2^x \cdot \frac{2^{0.001} - 1}{0.001}$$

$$= 2^x \cdot 0.69$$

$$f'(x) \doteq 0.69 \cdot 2^x$$

Minds On

Your Turn

We are going to work together to determine an expression for the derivative of

$$f(x) = b^x$$

We know that the derivative of $f(x) = 2^x$ is 0.69*2^x. We will refer to the constant 0.69 as *k*.

Determine the value of k for your assigned values of b and fill in the table below.

b	0.0001	0.001	0.01	0.1	0.25	0.5	0.75	1
k								

b	2	3	4	5	6	7	8	9
k	0.69							

$f(x) = b^x \qquad f'(x) = k \cdot b^x$

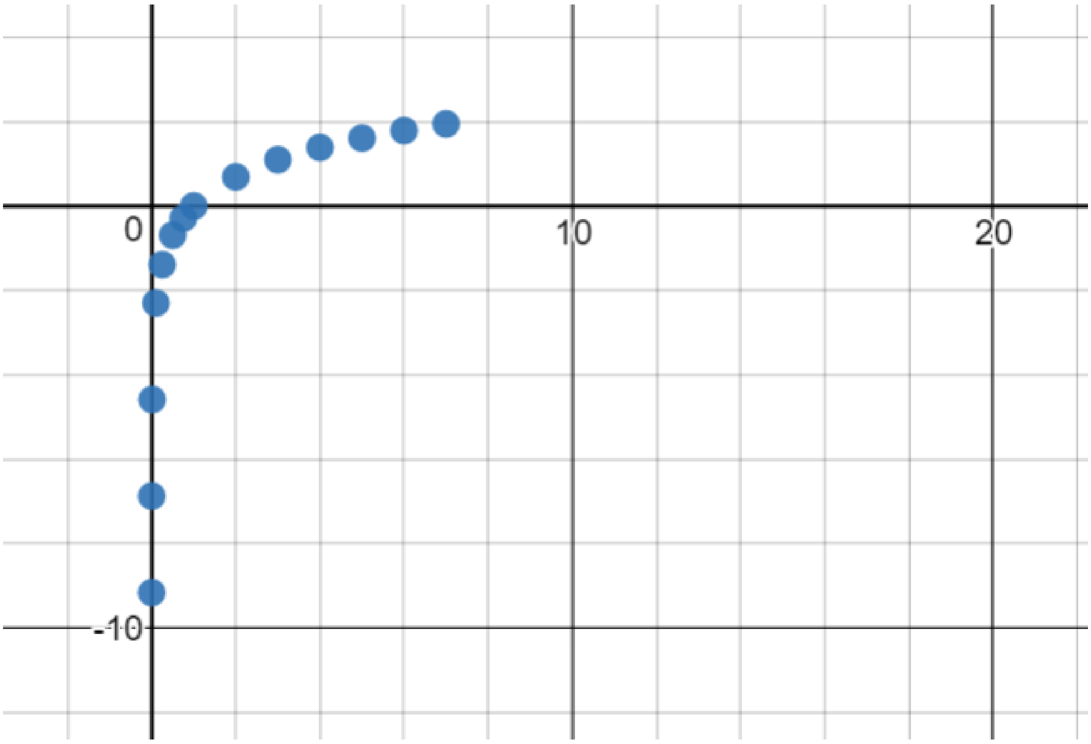
b	k
0.0001	-9.17
0.001	-6.96
0.01	-4.59
0.1	-2.30
0.25	-1.39
0.5	-0.69
0.75	-0.29
1	0

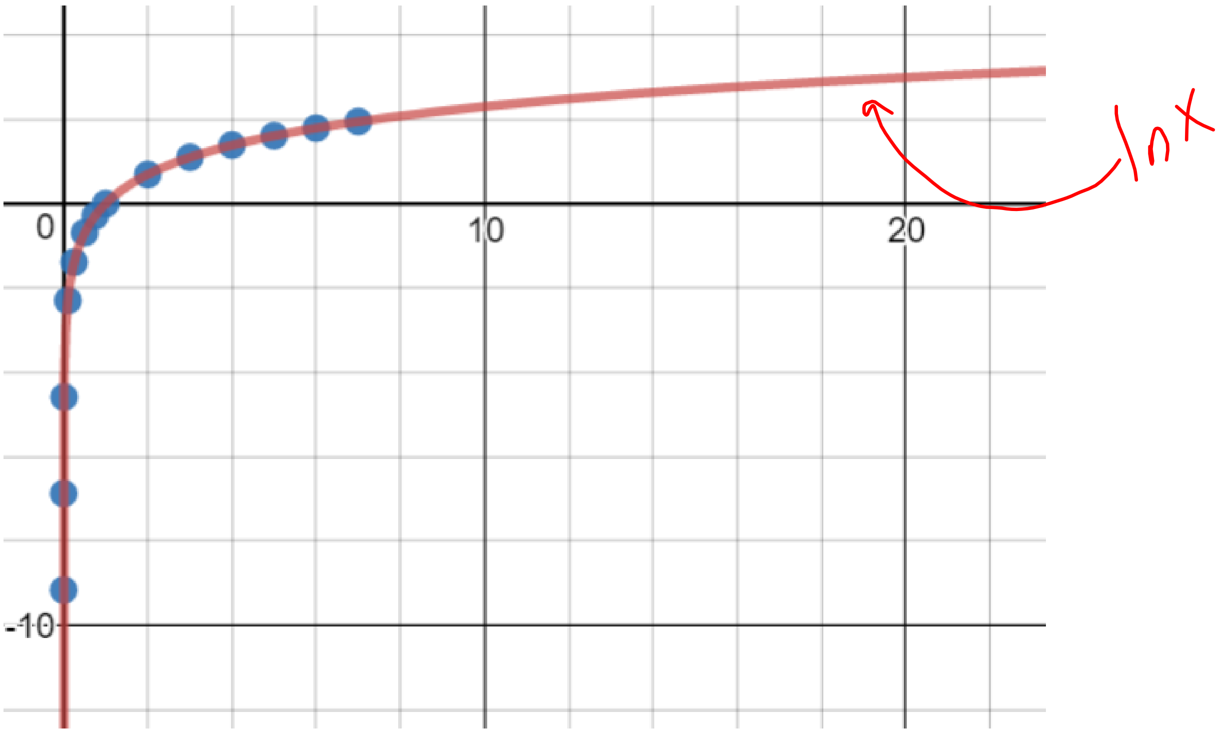
$f(x) = b^x$ $f'(x) = k \cdot b^x$

	b	k	
	2	0.69	
LG	3	1.10	0.41
JE	4	1.39	0.29
MA	5	1.61	0.22
JS	6	1.79	0.18
VG	7	1.95	0.16

We plotted (b, k) and it "looks like ln"

it is!





When $f(x) = b^x$

$f'(x) = \cancel{k} \cdot b^x$

$= \ln b \cdot b^x$

this means

$$\lim_{h \to 0} \frac{b^h - 1}{h} = \ln(b)$$

Action

5.2 The Derivative of the General Exponential Function

Key Ideas

- If $f(x) = b^x$, then $f'(x) = b^x \times \ln b$
- If $f(x) = b^{g(x)}$, then $f'(x) = b^{g(x)} \times \ln b \times g'(x)$
- $\lim_{h \to 0} \frac{b^h - 1}{h} = \ln b$ → From investigation
- When you are differentiating a function that involves an exponential function, use the rules given above, along with the sum, difference, product, ~~quotient~~, and chain rules as required.

Action

Example 1: Determine the derivatives of

a) $f(x) = 5^x$

$f'(x) = (5^x)(\ln 5) = 5^x \ln 5$

or

$= (\ln 5)(5^x) = (\ln 5)5^x$

b) $f(x) = 5^{3x-2}$

$f'(x) = 5^{3x-2} \cdot \ln(5) \cdot 3$

$= 3(5^{3x-2})\ln 5$

c) $\boxed{3^x}\boxed{x^6}$

$f'(x) = 3^x \cdot \ln 3 \cdot x^6 + 3^x \cdot 6x^5$

$= x^5 \cdot 3^x (x \cdot \ln 3 + 6)$

Action

Example 2: On January 1, 1850, the population of Weaverville was 50 000. The size of the population since then can be modelled by the function $P(t) = 50\,000(0.98)^t$, where t is the number of years since January 1st, 1850.

a) What was the population of Weaverville on January 1, 1900?

$t = 50$

$P(50) = 50000(0.98)^{50}$
$= 18,208$

b) At what rate was the population of Weaverville changing on January 1, 1900? Was it increasing or decreasing at that time?

$P'(t) = 50000 \times 0.98^t \times \ln 0.98$

$P'(50) \doteq -368$

Therefore, the population was decreasing at a rate of 368 people per year.

Consolidation

Based on **today's lesson** about derivatives of exponential functions, determine the derivative of **f(x) = e^x^**.

$f'(x) = e^x$

based on today

$\ln e = 1$

$f(x) = e^x$

$f'(x) = e^x \cdot \ln e$

$= e^x \cdot 1$

$= e^x$