Learning Goal: I will be able to solve optimization problems involving exponential function.

Minds On: Exploring Exponential Functions

Action: Optimizing Exponential Functions

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Minds On

Solve

$\log 2^x$

$\log_{10} 2^x = \log_{10} 17$

$\log 2^x = \log 17$

$\dfrac{x \log \cancel{2}}{\cancel{\log 2}} = \dfrac{\log 17}{\log 2}$

$x \doteq 4.087$

$2^x = 17$

$\ln 2^x = \ln 17$

$x = \dfrac{\ln 17}{\ln 2}$

$x \doteq 4.087$

$\boxed{\ln e = 1}$

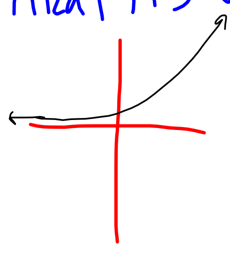
Minds On

The Optimization Algorithm

Today we are looking at optimization.

Before we get into optimizing exponential functions, let's recall the "optimization algorithm" that we have used previously.

- find "vertexes" (peaks & valleys)
  $f'(x) = 0$
- find endpoints
- sub critical #s & endpoints into $f(x)$

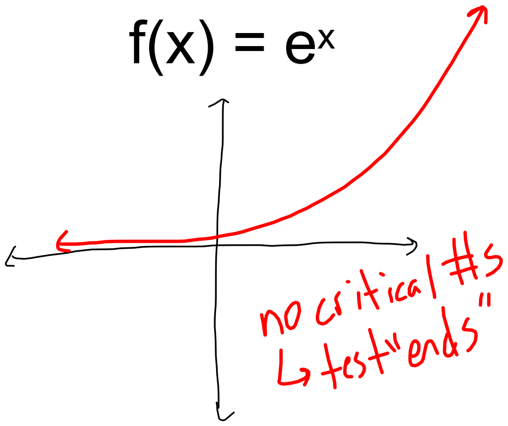


Minds On

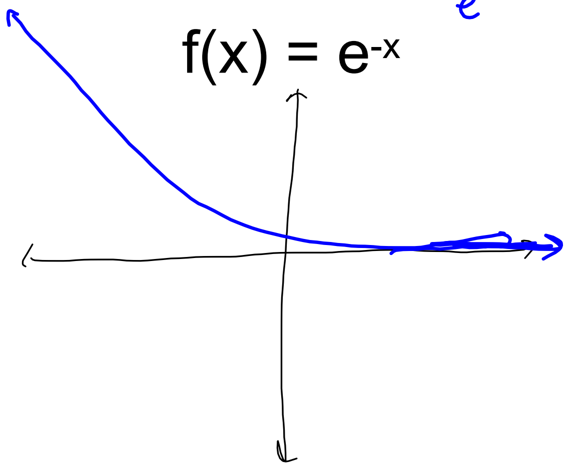
Exploring Exponential Functions

With optimization in mind, graph the following functions:

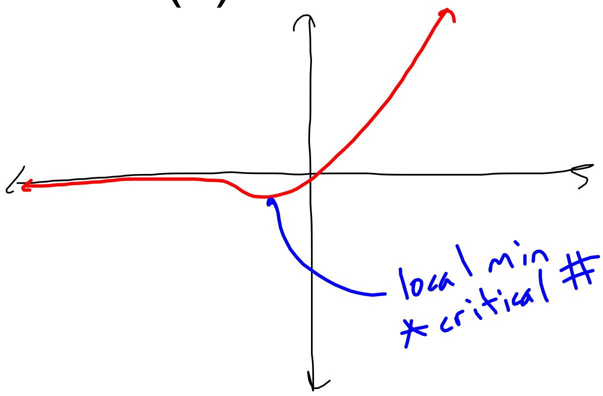
$f(x) = e^x$



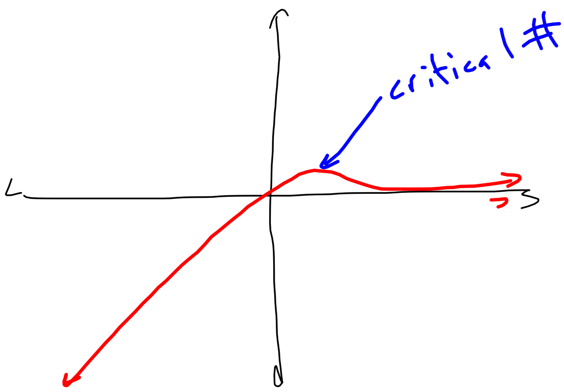
$f(x) = e^{-x}$ $\frac{1}{e^x}$



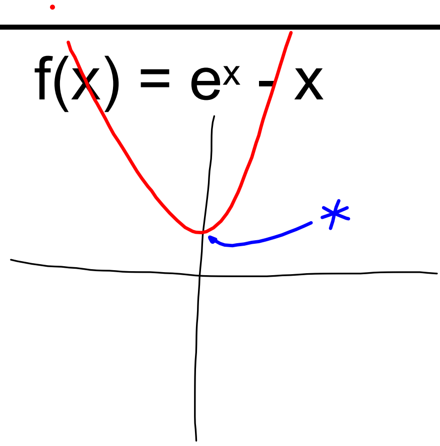
$f(x) = xe^x$



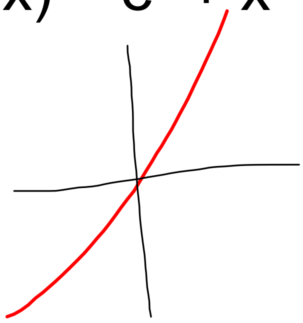
$f(x) = xe^{-x}$



$f(x) = e^x - x$



$f(x) = e^x + x$



Action

5.3 Optimization Problems Involving Exponential Functions

Example 1: The effectiveness of studying for an exam depends on how many hours a student studies. Some experiments show that if the effectiveness, E, is put on a scale of 0 to 10, then

$E(t) = 0.5[10 + te^{-\frac{t}{20}}]$, where t is the number of hours spent studying for an examination. If a student has up to 30 h for studying, how many hours are needed for maximum effectiveness?

Find critical points

not needed ~~unnecessary~~

$E(t) = 5 + 0.5te^{-\frac{t}{20}}$

$E'(t) = 0 + 0.5e^{-t/20} + 0.5t \cdot e^{-t/20} \cdot \cancel{\ln e} \cdot -\frac{1}{20}$

$= 0.5e^{-\frac{t}{20}} - \frac{1}{20} \cdot 0.5te^{-t/20}$

$= 0.5e^{-\frac{t}{20}}\left(1 - \frac{t}{20}\right)$

$= \dfrac{0.5(1 - t/20)}{e^{t/20}}$

$0 = \dfrac{0.5\left(1 - \frac{t}{20}\right)}{e^{t/20}}$

$$0 = \frac{0.5\left(1 - \frac{t}{20}\right)}{\cancel{e^{t/20}}}$$

$$0 = \cancel{0.5}\left(1 - \frac{t}{20}\right)$$

$$0 = 1 - \frac{t}{20}$$

$$\frac{t}{20} = 1$$

$$t = 20$$

When $t = 20$,

$$E(t) = 0.5\left[10 + 20e^{-\frac{20}{20}}\right]$$

$$= 9.64$$

$E(0) = 5$

$E(30) = 8.35$

$\therefore$ maximum effectiveness is reached with 20hrs of studying.

Action

Example 2: A mathematical consultant determines that the proportion of people who will have responded to the advertisement of a new product after it has been marketed for t days is given by $f(t) = 0.7(1 - e^{-0.2t})$. The area covered by the advertisement contains 10 million potential customers, and each response to the advertisement results in revenue to the company of \$0.70 (on average), excluding the cost of advertising. The advertising costs \$30 000 to produce and a further \$5000 per day to run.

a) Determine $\lim_{t\to\infty} f(t)$, and interpret the result.

b) What percent of potential customers have responded after seven days of advertising?

c) Write the function $P(t)$ that represents the average profit after t days of advertising. What is the average profit after seven days?

d) For how many full days should the advertising campaign be run in order to maximize the average profit? Assume an advertising budget of \$200 000.

a) $\lim_{t\to\infty} f(t) = \lim_{t\to\infty} 0.7(1-e^{-0.2t})$

$= 0.7 \times \lim_{t\to\infty} (1-e^{-0.2t})$

approaches 0

$= 0.7 \times 1$

$= 0.7$

70% of people will respond

b) $f(7) = 0.53$

53% have responded

c) Profit = revenue − cost

$$= \underbrace{\underset{\text{\$ per response}}{0.70} \times \underset{\text{number of responses}}{10{,}000{,}000}}\left(0.7\left(1-e^{-0.2t}\right)\right) - \left(30000 + 5000t\right)$$

$$= 7{,}000{,}000\left(0.7\left(1-e^{-0.2t}\right)\right) - 30000 - 5000t$$

$$= 4{,}900{,}000\left(1-e^{-0.2t}\right) - 30000 - 5000t$$

$$= 4{,}900{,}000 - 4{,}900{,}000e^{-0.2t} - 30000 - 5000t$$

$$= 4{,}870{,}000 - 4{,}900{,}000e^{-0.2t} - 5000t$$

$P(7) = 3{,}626{,}674.88$

d) For how many full days should the advertising campaign be run in order to maximize the average profit? Assume an advertising budget of $200 000.

$\to cost = 30000 + 5000t$

$$P(t) = -4{,}900{,}000e^{-0.2t} - 5000t + 4{,}870{,}000$$

Min. # days

0

Max. # of days

$30000 + 5000t = 200000$

$t = 34$

$$P'(t) = -4{,}900{,}000e^{-0.2t} \times -0.2 - 5000$$
$$= 980{,}000e^{-0.2t} - 5000$$

When $P'(t) = 0$

$$980{,}000e^{-0.2t} - 5000 = 0$$
$$980{,}000e^{-0.2t} = 5000$$
$$\frac{\cancel{980{,}000}e^{-0.2t}}{\cancel{980{,}000}} = \frac{5000}{980{,}000}$$
$$\ln e^{-0.2t} = \ln 0.0051$$
$$-0.2t \cancel{\ln e}^{1} = \ln 0.0051$$
$$-0.2t = \ln 0.0051$$
$$t = \frac{\ln 0.0051}{-0.2}$$
$$t = 26$$

Test $P(0), P(26), P(34)$

$P(0) = -30{,}000$

$\boxed{P(26) = 4{,}712{,}968.83}$

$P(34) = 4{,}694{,}542.50$

Consolidation

The Obliteration Situation

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