Learning Goal: I will be able to calculate derivatives of sinusoidal functions.

Minds On: Use your knowledge from curve sketching...

Action: Class note + practice

Consolidation: Exit Question

Minds On

Use your knowledge from curve sketching:

Given f(x) = sin x, sketch a graph of the derivative

OR

Given f(x) = cos x, sketch a graph of the derivative

Action

5.4 Derivatives of sinx and cosx

Derivatives of Sinusoidal Functions

$\frac{d}{dx}(sinx) = cosx$ $\frac{d}{dx}(cosx) = -sinx$

Example 1: Determine the derivative for each function.

a) y = cos3x

$\frac{dy}{dx} = -\sin 3x \cdot 3$

$= -3\sin 3x$

b) y = xsinx

$\frac{dy}{dx} = 1 \cdot \sin x + x \cos x$

$= \sin x + x \cos x$

c) y = $\sin x^2$

$y = \sin x^2$

$\frac{dy}{dx} = \cos x^2 \cdot 2x$

$= 2x \cos x^2$

d) y = $\sin^2 x$ $= (\sin x)^2$

$\frac{dy}{dx} = 2(\sin x) \cdot \cos x$

$= 2 \sin x \cos x$

$= \sin 2x$ ☺

$(3x+1)^2$

$2(3x+1)^1 \cdot 3$

Action

Derivatives of Composite Sinusoidal Functions

If y = sinf(x), then $\frac{dy}{dx} = cosf(x) \times f'(x)$

If y = cosf(x) then $\frac{dy}{dx} = -sinf(x) \times f'(x)$

Action

Example 2: Determine the derivative for $y = \cos(1 + x^3)$

$$\frac{dy}{dx} = -\sin(1+x^3) \cdot 3x^2$$

$$= -3x^2 \sin(1+x^3)$$

Action

Example 3: Determine y' for $y = e^{\sin x + \cos x}$

$$\frac{dy}{dx} = e^{\sin x + \cos x} \cdot 1 \cdot e \cdot \cos x - \sin x$$

$$= e^{\sin x + \cos x}(\cos x - \sin x)$$ ☺

OR

$$= (\cos x - \sin x)e^{\sin x + \cos x}$$ ☺

Action

slope = y'

Example 4: Determine the equation of the tangent to the graph of $y = x\cos 2x$ at $x = \pi/2$.

$y = x\cos 2x$ *radian time

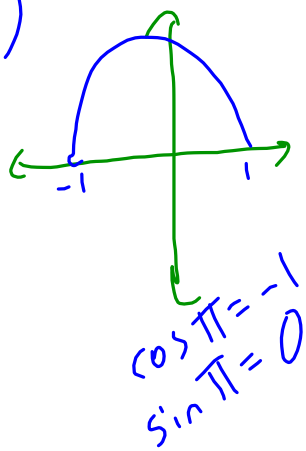
$y' = (1)\cos 2x + x(-\sin 2x)(2)$

$= \cos 2x - 2x\sin 2x$

when $x = \frac{\pi}{2}$ slope of tangent

$= \cos 2\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right)$

$= \cos\pi - \pi\sin\pi$



$\boxed{m = -1}$

$y = mx + b$

$b = y - mx$

$m = -1$

$x = \pi/2$

$y = \left(\frac{\pi}{2}\right)\cos 2\left(\frac{\pi}{2}\right)$

$= \frac{\pi}{2}\cos\pi$

$= -\frac{\pi}{2}$

$b = -\frac{\pi}{2} - (-1)\left(\frac{\pi}{2}\right)$

$b = -\frac{\pi}{2} + \frac{\pi}{2}$

$b = 0$

$\therefore y = -1x + 0$

$y = -x$

Action

Example 5: Determine the max and min values of the function $f(x) = \cos^2 x$ on the interval $[0, 2\pi]$

$f(x) = (\cos x)^2$

$f'(x) = 2\cos x \cdot -\sin x$

$= -2\cos x \sin x$

*We could rewrite as -2 sin 2x

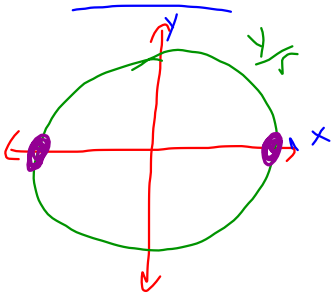
set $f'(x) = 0$

$\frac{0}{-2} = \frac{-2\cos x \sin x}{-2}$

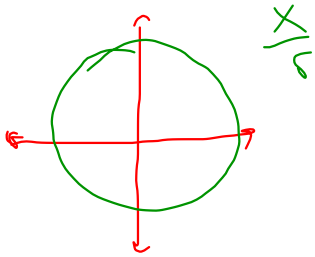
$\sin x \cos x = 0$

$\sin x \cos x = 0$ when $\sin x = 0$ and when $\cos x = 0$

$\sin x$

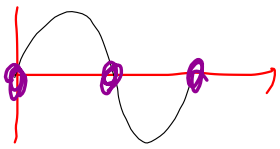


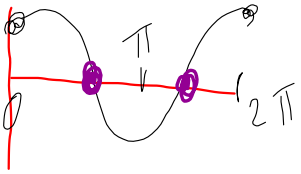
$\cos x$



$\sin x = 0$ when $x = 0, \pi, 2\pi$

$\cos x = 0$ when $x = \frac{\pi}{2}, \frac{3\pi}{2}$





Maxes and mins happen when $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

Find values:

$\cos^2 0 = 1$

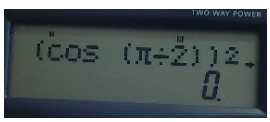
$\cos^2 \frac{\pi}{2} = 0$

$\cos^2 \pi = 1$

$\cos^2 \frac{3\pi}{2} = 0$

$\cos^2 2\pi = 1$

$* \cos^2 x = (\cos x)^2$



CONSOLIDATION
EXIT CARD

Page 256 #6a

$y' = 0$

6. Determine the absolute extreme values of each function on the given interval. (Verify your results with graphing technology.)

a. $y = \cos x + \sin x, 0 \le x \le 2\pi$

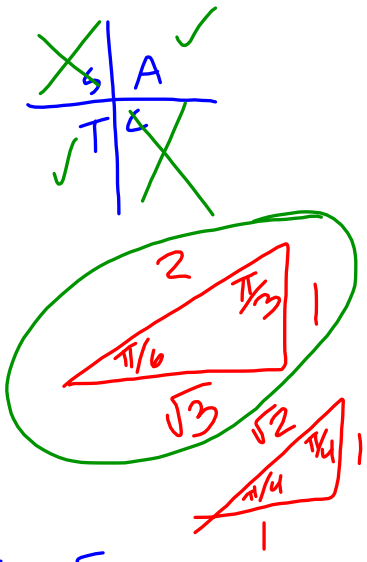
$y' = -\sin x + \cos x$

when $y' = 0$

$-\sin x + \cos x = 0$

$\cos x = \sin x$

when does $\cos x = \sin x$?



$\cos x = \sin x$ when $x = 45°$ $(\frac{\pi}{4})$ and $225°$ $(\frac{5\pi}{4})$

when $x = \frac{\pi}{4}$ $y = \cos\frac{\pi}{4} + \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$x = \frac{5\pi}{4}$ $y = \cos\frac{5\pi}{4} + \sin\frac{5\pi}{4}$

$= -\frac{1}{\sqrt{2}} + -\frac{1}{\sqrt{2}} = -\frac{2}{\sqrt{2}} = -\frac{2\sqrt{2}}{2} = -\sqrt{2}$