<u>Implicit Differentiation</u>

We're used to functions written as y = or f(x) = , where y is isolated on one side and is expressed explicitly in terms of our x variable. Many functions are defined implicitly, such as the circle $x^2 + y^2 = 16$.

Since there are x-values that correspond to two y-values, y is not a function of x on the entire circle. (fails the vertical line test)

To solve for y we would get y =

where   represents the upper semicircle and   the lower.

If we were asked to determine the slope of the tangent to the circle at the point (0,4), we would differentiate the function   and sub in x = 0. This process required us to

first rearrange the equation for y, then choose which part of the equation we were going to use, then differentiate the equation, then solve.

In this circle problem, we could have prepared to take the derivative, y', by first rearranging for y. There are many situations where solving for y in terms of x is very difficult, and sometimes impossible. In these cases, we use an alternative method called **<u>implicit differentiation</u>**.

To fully understand implicit differentiation, we need to remember/highlight a couple of things:

1. $\frac{dy}{dx}$ vs $\frac{d}{dx}$

2. How the chain rule comes into play:

<u>Example 1:</u> a) If $x^2 + y^2 = 25$, determine dy/dx.

b) Determine the slope of the tangent to the circle $x^2 + y^2 = 25$ at the point (3, -4).

Example 2: Determine dy/dx for $2xy - y^3 = 4$.

Procedure for Implicit Differentiation

If an equation defines y implicitly as a differentiable function of x, determine dy/dx as follows:

1. Differentiate both side of the equation with respect to x. Remember to use the chain rule when differentiating terms containing y.
2. Solve for dy/dx by rearranging.