Learning Goal: I will be able to solve problems involving related rates.

Minds On: d what? / d huh?

Action: Related Rates

Consolidation: Doubly Dependent

RAFT

Consolidation

Exit Question

Determine dy/dx for $(x + y)^3 = 12x$

$$\frac{d}{dx}(x+y)^3 = \frac{d}{dx}12x$$

$$3(x+y)^2 \cdot \frac{d}{dx}(x+y) = 12$$

$$3(x+y)^2 \cdot \left(\frac{d}{dx}x + \frac{d}{dx}y\right) = 12$$

$$3(x+y)^2\left(1 + \frac{dy}{dx}\right) = 12$$

$$1 + \frac{dy}{dx} = \frac{12}{3(x+y)^2}$$

$$\frac{dy}{dx} = \frac{4}{(x+y)^2} - 1$$

Minds On

$\frac{d\ what?}{d\ huh?}$

Yesterday we kept trying to find dy/dx (change in y wrt x).

In real world problems (excluding those involving yaks, yurts, x-rays, and xylophones) y and x aren't particularly relevant.

For each scenario, determine what d_/d_ you are looking for.

1. Determine the rate at which the radius of a circle is increasing over time. $\frac{dr}{dt}$

2. Determine the rate of increase of the circumference of a circle with respect to time. $\frac{dC}{dt}$

3. Determine the rate of decrease of the area of a circle with respect to time. $\frac{dA}{dt}$

3. At what rate are McKenna and Logan's vehicles separating 3 minutes after they leave the school?

$\frac{dD}{dt}$

4. How fast is the height of the rocket increasing 10 minutes after launch? $\frac{dh}{dt}$

5. The radius of a tree is increasing by 0.003 m per year, the height is increasing by 0.4 m per year. Determine the rate of increase of the volume of the trunk of the tree when the diameter is 1 m and the height is 15 m.

$\frac{dV}{dt}$

Minds On

Oil Spill

Oil that is spilled from a tanker spreads in a circle. The area of the circle increases at a constant rate of 6 km^2/h.

Will the radius be increasing at the same rate at all times? ie: Will it take the same amount of time for the radius to increase from 1 km to 2 km as it does to increase from 2 km to 3 km? Discuss with someone.

Action

This section on related rates focuses on understanding the applications of derivatives and how they can be used to describe and predict change. In many real-life applications, several quantities vary in relation to one another. Not only do the quantities vary in relation to each other, but the rates at which they vary are related to one other ... so we have **related rates**.

Action

Example 1: When a raindrop falls into a still puddle, it creates a circular ripple that spreads out from the point where the raindrop hit. The radius of the circle grows at a rate of 3 cm/s. $\frac{dr}{dt}$

a) Determine the rate of increase of the circumference of the circle with respect to time.

b) Determine the rate of increase of the area $\frac{dA}{dt}$ of the circle when its area is 81π cm^2. A

a) $C = 2\pi r$

$\frac{d}{dt}C = \frac{d}{dt}2\pi r$

$\frac{dC}{dt} = \frac{d}{dr}2\pi r \times \frac{dr}{dt}$

$\frac{dC}{dt} = 2\pi \times \frac{dr}{dt}$

$\frac{dC}{dt} = 6\pi$ cm/s

b) $A = \pi r^2$

$\frac{dA}{dt} = \frac{d}{dt}\pi r^2$

$\frac{dA}{dt} = \frac{d}{dr}\pi r^2 \times \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi r \times \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(9)(3)$

$\frac{dA}{dt} = 54\pi$ cm^2/s

$A = \pi r^2$

$81\pi = \pi r^2$

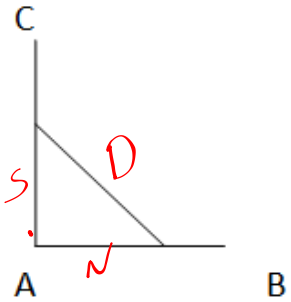
$81 = r^2$

$r = 9$

Action

Example 2: Natalie and Shannon start from point A and drive along perpendicular roads AB and AC respectively, as shown. Natalie drives at a speed of 45 km/h, and Shannon travels at a speed of 40 km/h. If Shannon begins 1 h before Natalie, at what rate are their cars separating 3 h after Shannon leaves?

$\frac{dD}{dt}$



$$D^2 = S^2 + N^2$$

$$\frac{d}{dt}D^2 = \frac{d}{dt}S^2 + \frac{d}{dt}N^2$$

$$\frac{d}{dD}D^2 \times \frac{dD}{dt} = \frac{d}{dS}S^2 \times \frac{dS}{dt} + \frac{d}{dN}N^2 \times \frac{dN}{dt}$$

$$2D\frac{dD}{dt} = 2S\frac{dS}{dt} + 2N\frac{dN}{dt}$$

$$D\frac{dD}{dt} = S\frac{dS}{dt} + N\frac{dN}{dt}$$

$S = 120$

$N = 90$

$\frac{dS}{dt} = 40$

$\frac{dN}{dt} = 45$

$D = \sqrt{120^2 + 90^2}$

$D = 150$

$$150\frac{dD}{dt} = (120)(40) + (90)(45)$$

$$\frac{dD}{dt} = 59 \text{ km/h}$$

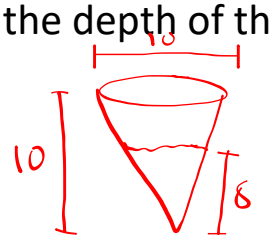
Action $\frac{dV}{dt}$

Example 3: Water is pouring into an inverted cone at a rate of π m^3/min. The height and diameter of the base of the cone are both 10 m. How fast is the water level rising when the depth of the water is 8 m?

h + d

h

? → $\frac{dh}{dt}$



$h = d$
$h = 2r$
$r = \frac{h}{2}$

$$V = \frac{1}{3}\pi r^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$$

$$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)(h)$$

$$V = \frac{1}{12}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{12}\pi \times \frac{d}{dt}h^3$$

$$\frac{dV}{dt} = \frac{1}{12}\pi \times \frac{d}{dh}h^3 \times \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{1}{12}\pi \times 3h^2 \times \frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{1}{4}\pi h^2 \times \frac{dh}{dt}$$

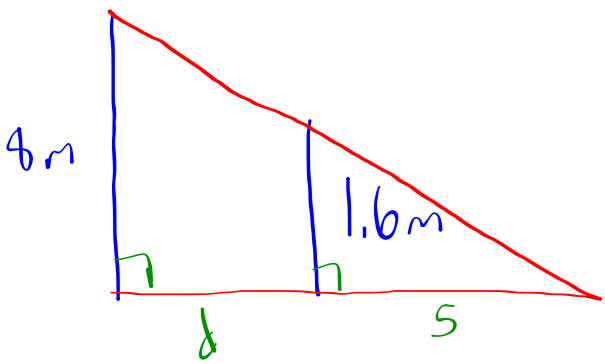
$$\frac{dh}{dt} = \frac{\frac{dV}{dt}}{\frac{1}{4}\pi h^2}$$

$$\frac{dh}{dt} = \frac{\cancel{\pi}}{\frac{1}{4}\cancel{\pi}(4)^2}$$

$$\frac{dh}{dt} = \frac{1}{\frac{64}{4}} = \frac{1}{16} \text{ m/min}$$

Action

Example 4: A student who is 1.6 m tall walks directly away from an 8 m tall lamppost at a rate of 1.2 m/s. Determine the rate at which the shadow is lengthening when she is 20 m from the base of the lamppost.



$$\frac{8}{1.6} = \frac{d+s}{s}$$

$$8s = 1.6(d+s)$$

$$8s = 1.6d + 1.6s$$

$$0 = 1.6d + 1.6s - 8s$$

$$0 = 1.6d - 6.4s$$

$$\frac{6.4s}{1.6} = \frac{1.6d}{1.6}$$

$$\boxed{4s = d}$$

$d = 4s$

$$d = 4s$$

$$\frac{d}{dt} d = \frac{d}{dt} 4s$$

$$\frac{dd}{dt} = \frac{d\,4s}{ds} \times \frac{ds}{dt}$$

$$\frac{dd}{dt} = 4 \times \frac{ds}{dt}$$

$$1.2 = 4 \times \frac{ds}{dt}$$

$$\frac{ds}{dt} = 0.3\,m/s$$

Consolidation

Doubly Dependent

What are some examples of things that depend on things that depend on other things?

- Foot size depends on height, and height depends on age.