Learning Goal: I will be able to solve problems involving vector addition.

Minds On: Tugboats

Action: 1. Class Examples
2. Practice on page 290

Consolidation: Exit Question

Minds On

Paper Toss (5 trials)

For this investigation we are going to need to ignore gravity to keep things 2 dimensional.

We need a paper ball thrower.

We need an impact marker.

We two measurers.

We need four teams of mathematicians.

The goal: determine the speed at which the thrower throws the ball (pretend we exist in 2 dimensions)

Throw "Speed" (m/s)

Toss	Group 1	Group 2	Group 3
1	~~6.94~~	5.02	4.97
2	~~6.06~~	5.44	5.64
3	5.29	4.44	~~6.30~~
4	6.04	5.43	~~6.70~~
5	~~4.31~~	6.07	5.69

"Average Speed" = 5.44 m/s

Emily's Throw = 5.44m/s

Throwing	Group 1	Group 2	Group 3
N	~~3.69~~	4.51	4.76
S	~~3.95~~	5.25	4.94
E	4.96	4.21	~~6.26~~
W	6.54	~~6.40~~	6.94

Minds On

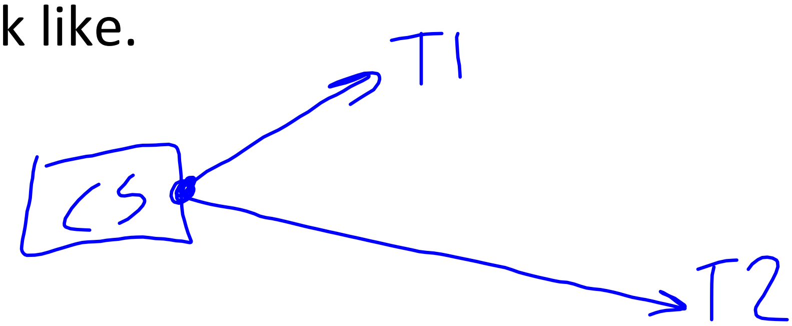
Paper Toss Part 2

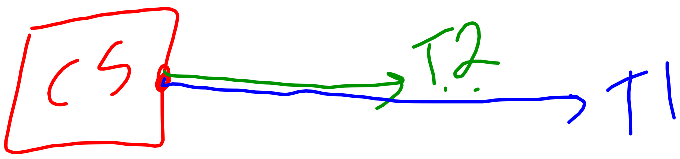
Now we will go outside and do it again.

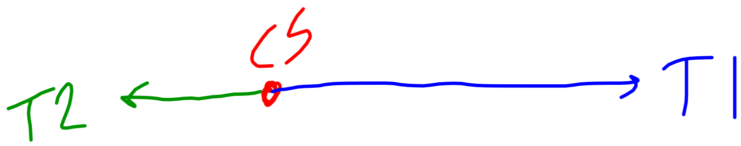
Minds On

Suppose that a cargo ship has a mechanical failure and must be towed into port by 2 tugboats.

1. Draw a picture (or pictures) of what this might look like.



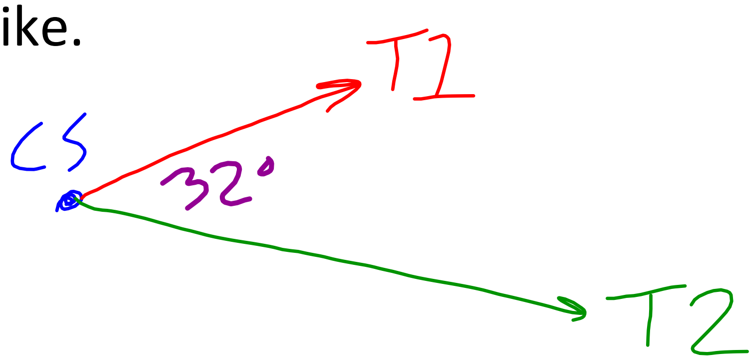




Minds On

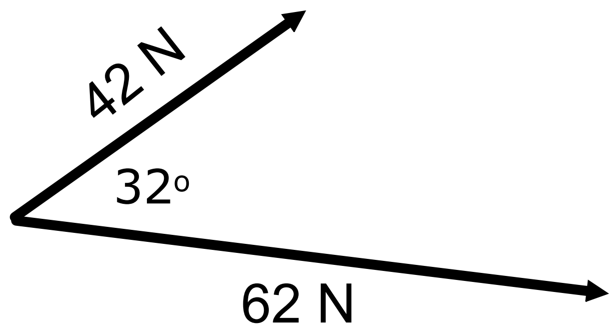
Suppose that a cargo ship has a mechanical failure and must be towed into port by 2 tugboats.

1. Draw a picture (or pictures) of what this might look like.



2. Now, suppose you're given this additional information: the tugboats are pulling at an angle of 32° to each other. Can you add to your picture or draw a new picture to show what direction you think the cargo ship will travel?

3. Suppose you're told that one tugboat pulls with a force of 42 N and the other pulls with a force of 62 N. Can you find the force and direction that the cargo ship is moving in?

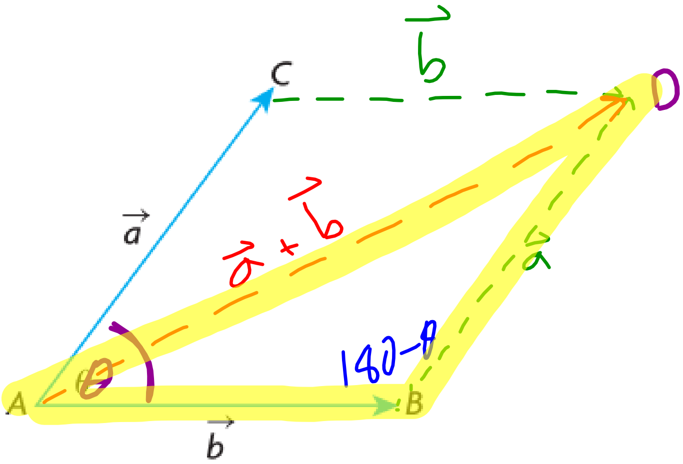


Action

6.2 Vector Addition

When we are presented with vectors that are working together (or against each other), we have to find ways to combine these vectors into a single vector that shows the outcome. This is called the ***RESULTANT*** vector. We can find the resultant vector in one of two ways:

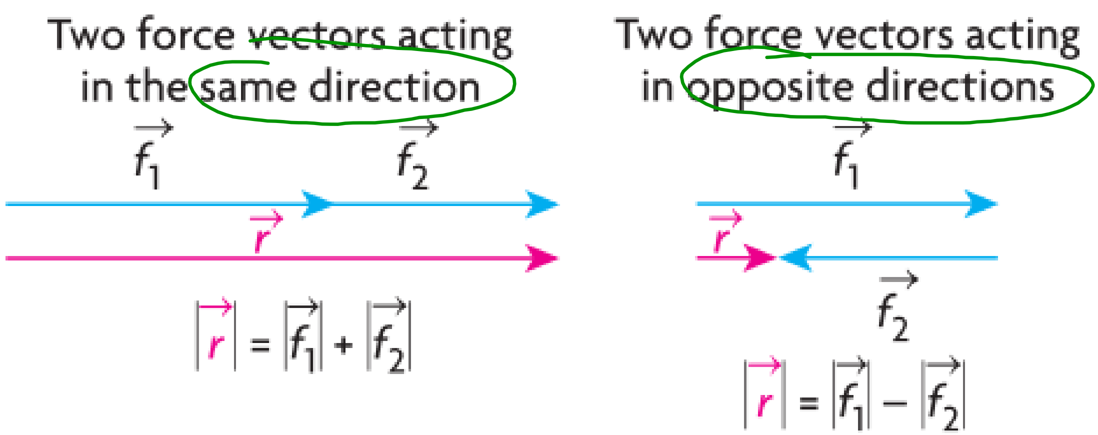
1. The Parallelogram Law for Vector Addition



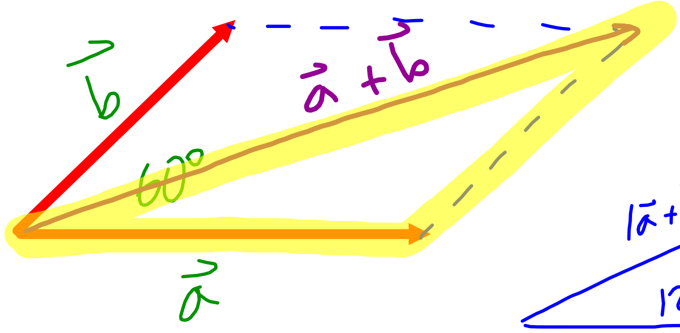
To determine the sum of the two vectors $\vec{a}$ and $\vec{b}$, complete the parallelogram formed by these two vectors when placed tail to tail. Their sum is the vector $\overrightarrow{AD}$, the diagonal of the constructed parallelogram,

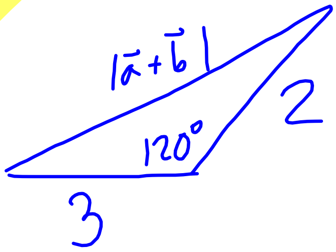
$\vec{a} + \vec{b} = \overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$.

Consider the triangle formed by vectors $\vec{a}$, $\vec{b}$ and $\vec{a} + \vec{b}$. It is important to note that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$. This means that the magnitude of the sum $\vec{a} + \vec{b}$ is less than or equal to the combined magnitudes of $\vec{a}$ and $\vec{b}$. The magnitude of $\vec{a} + \vec{b}$ is equal to the sum of the magnitudes of $\vec{a}$ and $\vec{b}$ only when these three vectors lie in the same direction.

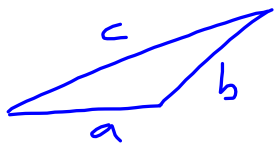


Example 1: Given vectors $\vec{a}$ and $\vec{b}$ such that the angle between the two vectors is 60°, $|\vec{a}| = 3$ and $|\vec{b}| = 2$, determine $|\vec{a} + \vec{b}|$.





Cosine Law



$c^2 = a^2 + b^2 - 2ab\cos C$

$|\vec{a}+\vec{b}|^2 = 3^2 + 2^2 - 2(3)(2)\cos 120°$

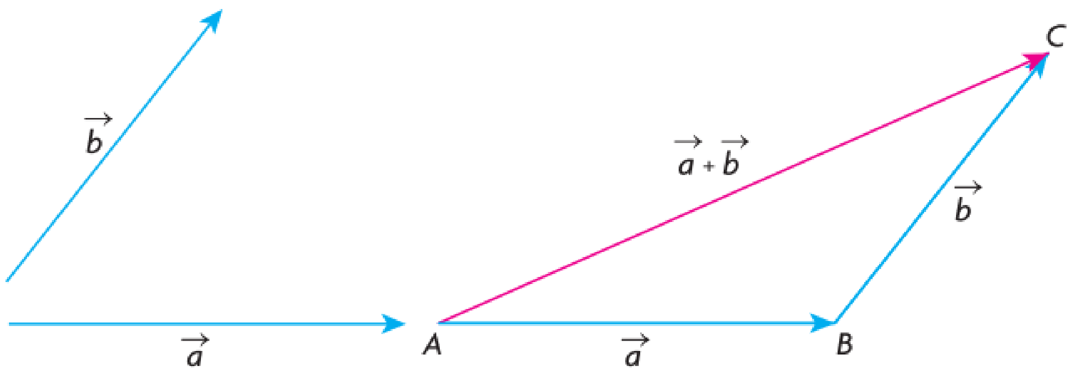
$|\vec{a}+\vec{b}|^2 = 9 + 4 - (-6)$

$|\vec{a}+\vec{b}|^2 = 19$

$|\vec{a}+\vec{b}| \doteq 4.36$

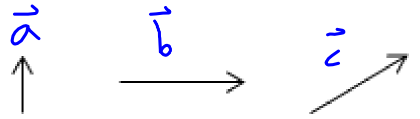
2. The Triangle Law for Vector Addition

Triangle Law of Addition

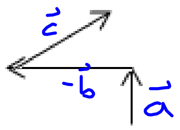


In the diagram, the sum of the vectors $\vec{a}$ and $\vec{b}$, $\vec{a} + \vec{b}$, is found by translating the tail of vector $\vec{b}$ to the head of vector $\vec{a}$. This could also have been done by translating $\vec{a}$ so that its tail was at the head of $\vec{b}$. In either case, the sum of the vectors $\vec{a}$ and $\vec{b}$ is $\overrightarrow{AC}$.

Example 2: Use the three vectors below to sketch $\vec{a} - \vec{b} + \vec{c}$.

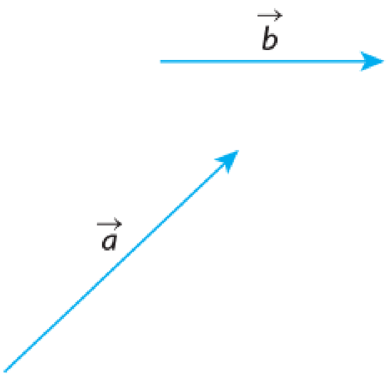


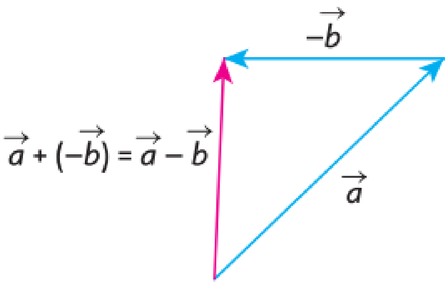
Same as $\vec{a} + (-\vec{b}) + \vec{c}$



What can you conclude about how to find the difference between vectors?

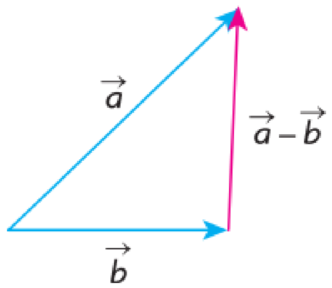
The Difference of Two Vectors, $\vec{a} - \vec{b}$



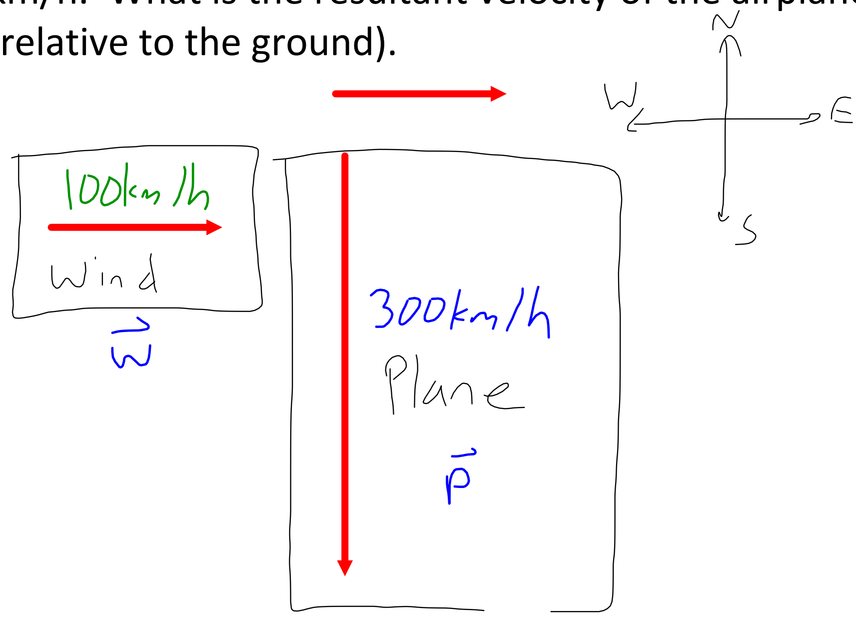


In the diagram above, the difference between vectors $\vec{a}$ and $\vec{b}$ is found by adding the opposite vector $\vec{b}$ to $\vec{a}$ using the triangle law of addition.

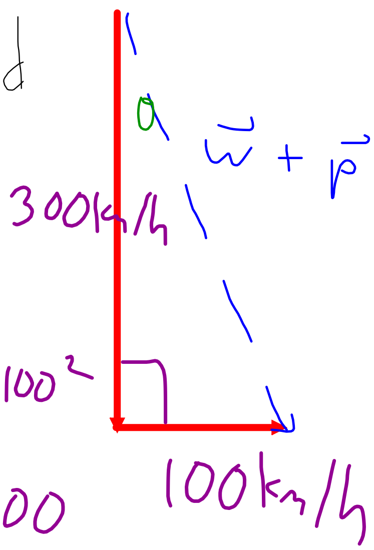
Another way to think about $\vec{a} - \vec{b}$ is to arrange the vectors tail to tail. In this case, $\vec{a} - \vec{b}$ is the vector that must be added to $\vec{b}$ to get $\vec{a}$. This is illustrated in the following diagram. Using the vectors above, the difference vector is the same as the one produced by adding the opposite.



Example 3: An airplane heads due south at a speed of 300 km/h and meets a wind from the west at 100 km/h. What is the resultant velocity of the airplane? (relative to the ground).



Resultant: Add



$|\vec{w}+\vec{p}|^2 = 300^2 + 100^2$

$|\vec{w}+\vec{p}|^2 = 100,000$

$|\vec{w}+\vec{p}| = 316.2 \text{ km/h}$

Angle

$\tan\theta = \frac{100}{300} = \frac{1}{3}$

$\theta = 18.4^\circ$

∴ plane is flying at a speed of 316.2 km/h in a direction of S 18.4° E

or

"18.4° East of South"

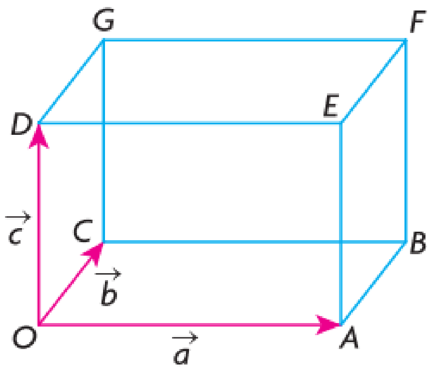
or

in a south direction, 18.4° toward the east

Consolidation

Representing a single vector as a combination of vectors

In the rectangular box shown below, $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OC} = \vec{b}$, and $\overrightarrow{OD} = \vec{c}$.



Express each of the following vectors in terms of $\vec{a}$, $\vec{b}$, and $\vec{c}$.

a. $\overrightarrow{BC}$ b. $\overrightarrow{GF}$ c. $\overrightarrow{OB}$ d. $\overrightarrow{AC}$ e. $\overrightarrow{BG}$ f. $\overrightarrow{OF}$

Consolidation

Homework is on page 290-292

EXIT QUESTION

page 291 #7
Complete on your own, then check with a partner.
I can clarify any arguments!