6.4 Properties of Vectors

Officially, here are the first 3 specific rules for dealing with vectors:

Commutative Property of Addition

When we are dealing with numbers, the order in which they are added does not affect the final answer. With vectors, addition follows the same rules we first learned with addition and with algebra:

Algebraically, the **Commutative Property of Addition** is:

There are other vector operations that are not commutative – watch out for these later on and make sure you keep your rules straight!

Associative Property of Addition

When adding numbers, we are free to associate the numbers any way we choose. With numbers and with algebra it looks like this:

Algebraically, the **Associative Property of Addition** is:

As with the commutative property, the associative property does not hold true for all operations.

Distributive Property of Addition

When we have a scalar in front of a bracket, the scalar must be multiplied by each term in the bracket. With algebra it looks like this:

Algebraically, the **Distributive Property of Addition** is:

This property depends on the properties of similar triangles.

Example 1: ***Simplify*** the following expression:

$3(2\vec{a} + \vec{b} + \vec{c}) - (\vec{a} + 3\vec{b} - 2\vec{c})$

Further Laws of Vector Addition and Scalar Multiplication

Adding the zero vector: $\vec{a} + \vec{0} = \vec{a}$

Associative Law for Scalars: $m(n\vec{a}) = (mn)\vec{a} = mn\vec{a}$

Distributive Law for Scalars: $(m + n)\vec{a} = m\vec{a} + n\vec{a}$

Example 2: If $\vec{x} = 3\vec{\imath} - 4\vec{\jmath} + \vec{k}$, $\vec{y} = \vec{\jmath} - 5\vec{k}$, and $\vec{z} = -\vec{\imath} - \vec{\jmath} + 4\vec{k}$ determine each of the following in terms of $\vec{\imath}$, $\vec{\jmath}$, and $\vec{k}$.

a) $\vec{x} + \vec{y}$

b) $\vec{x} - \vec{y}$

c) $\vec{x} - 2\vec{y} + 3\vec{z}$