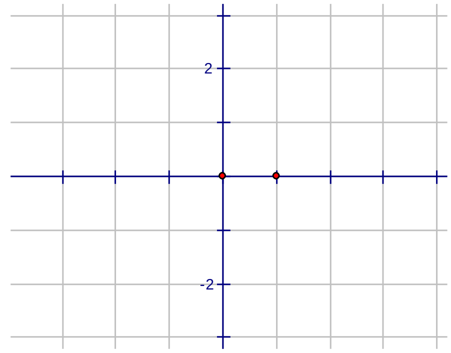
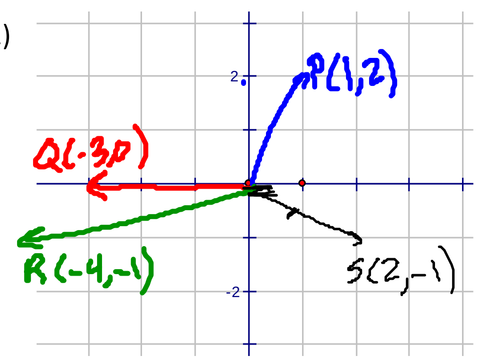
<u>6.6 Operations with Algebraic Vectors in R^2</u>

Last day, we wrote that the vector that started at the origin and ended at P(a, b) was called $\overrightarrow{OP} = (a, b)$. A second way of writing this is with the use of the unit vectors $\vec{\imath}$ and $\vec{\jmath}$. The vectors $\vec{\imath} = (1, 0)$ and $\vec{\jmath}$ = (0, 1) have magnitude 1 and lie on the x- and y-axes, respectively.



Example 1: a) Draw four position vectors, $\overrightarrow{OP}$ = (1,2), $\overrightarrow{OQ}$ = (-3, 0), $\overrightarrow{OR}$ = (-4, -1) and $\overrightarrow{OS}$ = (2, -1). Write each of these vectors using the unit vectors $\vec{\imath}$ and $\vec{\jmath}$.



$\overrightarrow{OP} = 1\vec{\imath} + 2\vec{\jmath}$ $\quad$ $\overrightarrow{OR} = -4\vec{\imath} - \vec{\jmath}$

$\overrightarrow{OQ} = -3\vec{\imath}$ $\quad$ $\overrightarrow{OS} = 2\vec{\imath} - \vec{\jmath}$

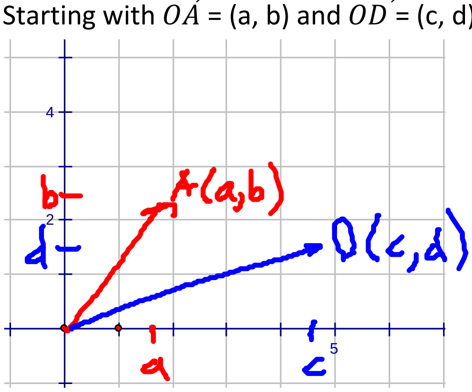
b) The vectors $\overrightarrow{OA} = -\vec{\imath}$, $\overrightarrow{OB} = \vec{\imath} + 5\vec{\jmath}$, $\overrightarrow{OC} = -5\vec{\imath} + 2\vec{\jmath}$ and $\overrightarrow{OD} = 2\vec{\imath} - 4\vec{\jmath}$ have been written using the unit vectors $\vec{\imath}$ and $\vec{\jmath}$. Write them in component form (a, b).

$\overrightarrow{OA} = (-1, 0)$ $\quad$ $\overrightarrow{OC} = (-5, 2)$

$\overrightarrow{OB} = (1, 5)$ $\quad$ $\overrightarrow{OD} = (2, -4)$

Addition of Two Vectors Using Component Form

Starting with $\overrightarrow{OA}$ = (a, b) and $\overrightarrow{OD}$ = (c, d) with A and D both on R^2, we can also write $\overrightarrow{OA} = a\vec{\imath} + b\vec{\jmath}$ and $\overrightarrow{OD} = c\vec{\imath} + d\vec{\jmath}$.



Add: $\overrightarrow{OA} + \overrightarrow{OD} = (a+c, b+d)$

$= (a+c)\vec{\imath} + (b+d)\vec{\jmath}$

Subtract: $\overrightarrow{OA} - \overrightarrow{OD} = (a-c, b-d)$

$= (a-c)\vec{\imath} + (b-d)\vec{\jmath}$

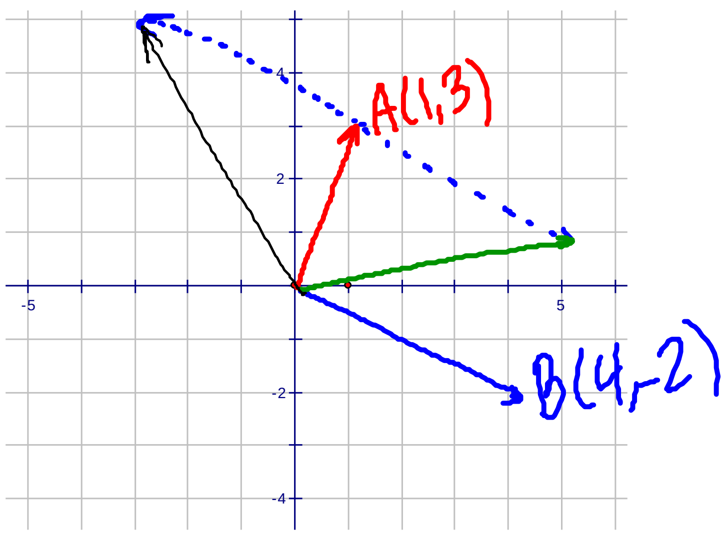
Scalar Multiplication of Vectors Using Components

When we want to multiply a scalar by a vector in component form, we must multiply each of the components by the scalar as follows:

$\overrightarrow{OP} = (a,b)$, we want $m\overrightarrow{OP}$

$$m\overrightarrow{OP} = m(a,b)$$
$$= m(a\vec{i} + b\vec{j}) = (ma)\vec{i} + (mb)\vec{j}$$
$$= (ma, mb)$$

Example 2: Given $\vec{a} = \overrightarrow{OA} = (1,3)$ and $\overrightarrow{OB} = \vec{b} = (4, -2)$, determine the components of $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, and illustrate each of these vectors on the graph.



$\boxed{\vec{a} + \vec{b} = (5, 1)}$

$\boxed{\vec{a} - \vec{b} = (-3, 5)}$

Position Vectors and Magnitudes in R^2

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, then the vectors $\overrightarrow{AB} = (x_2 - x_1, y_2 - y_1)$ is its related position vector $\overrightarrow{OP}$, and $|\overrightarrow{AB}| = \sqrt{\cancel{(x2 - x1)^2 + (y2 - y1)^2}}$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$

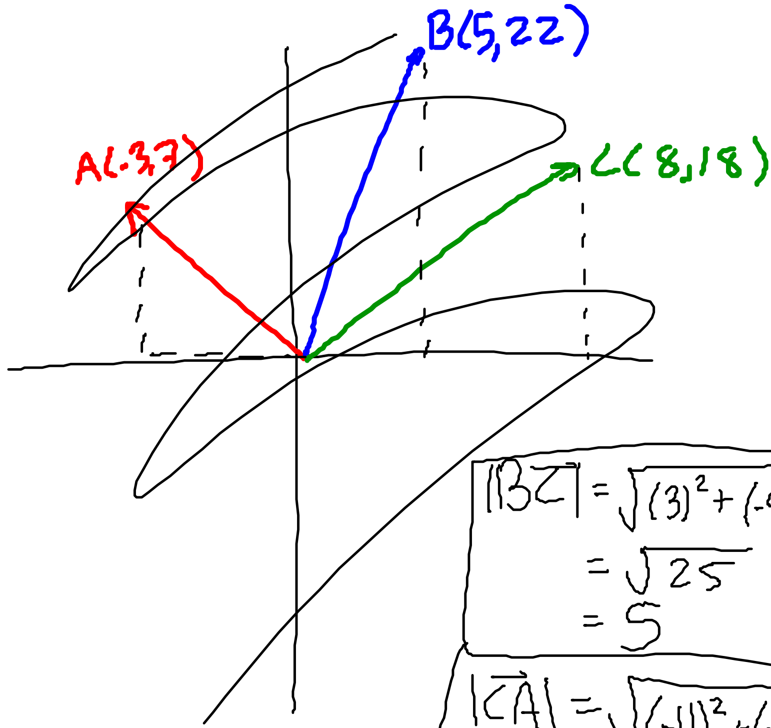
OB

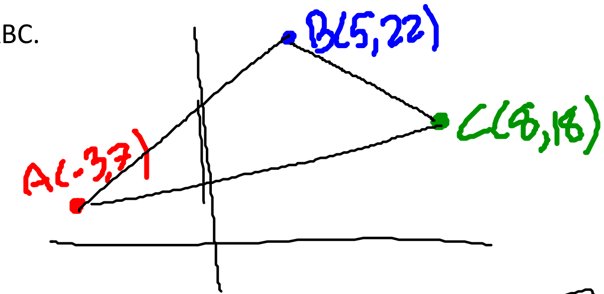
$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\overrightarrow{AB} = \vec{b} - \vec{a}$$

Example 3: A(-3, 7), B(5, 22), and C(8, 18) are three points in R^2.

a) Calculate the value of $|\overrightarrow{AB}| + |\overrightarrow{BC}| + |\overrightarrow{CA}|$, the perimeter of triangle ABC.





$$|\overrightarrow{AB}| = \sqrt{(5-(-3))^2+(22-7)^2}$$
$$= \sqrt{64+225}$$
$$= \sqrt{289}$$
$$= 17$$

$$|\overrightarrow{BC}| = \sqrt{(3)^2+(-4)^2}$$
$$= \sqrt{25}$$
$$= 5$$

$$|\overrightarrow{CA}| = \sqrt{(-11)^2+(-11)^2}$$
$$= \sqrt{242}$$
$$= 11\sqrt{2}$$

$$P = 22 + 11\sqrt{2}$$
$$P = 22 + \sqrt{242}$$

b) Calculate the value of $|\overrightarrow{AB} + \overrightarrow{BC}|$

$$= |\overrightarrow{AC}|$$
$$= \sqrt{242} \text{ or } 11\sqrt{2}$$

Example 4: For the vectors $\vec{x} = 2\vec{i} - 3\vec{j}$ and $\vec{y} = -4\vec{i} - 3\vec{j}$, determine $|\vec{x}+\vec{y}|$ and $|\vec{x}-\vec{y}|$.

$$\vec{x}+\vec{y} = (2\vec{i}-3\vec{j}) + (-4\vec{i}-3\vec{j})$$
$$= -2\vec{i} - 6\vec{j}$$
$$= (-2, -6)$$
$$|\vec{x}+\vec{y}| = \sqrt{(-2)^2+(-6)^2}$$
$$= \sqrt{40}$$
$$= 2\sqrt{10}$$

$$\vec{x}-\vec{y} = (2\vec{i}-3\vec{j}) - (-4\vec{i}-3\vec{j})$$
$$= 6\vec{i}$$
$$= (6, 0)$$
$$|\vec{x}-\vec{y}| = \sqrt{(6)^2+(0)^2}$$
$$= \sqrt{36}$$
$$= 6$$

duh! It's just: