Learning Goal: I will be able to use vectors to model and solve real-world problems involving velocity and force.

Minds On: 1. Force in real life
2. Resultant and Equilibrant

Action: 1. Class Examples
2. Practice on page 362

Consolidation: Exit Question

Minds On

RAFT

Read "A bit of your textbook" For Ten

Read page 352-353

Stop when you get to Example 1

Minds On

7.1 Vectors as Forces

We usually associate force with muscular exertion, such as pulling a sled, lifting a book, shooting a basketball, or pedalling a bicycle. There are example of force where muscular action is not present, such as the attraction of the Moon to Earth, the attraction of a magnet to the fridge, the thrust exerted by an engine when gas combusts in its cylinders, or the force exerted by shock absorbers to reduce vibration.

Force is defined as something that either pushes or pulls on an object. On Earth, force is the product of the mass of an object and the acceleration due to gravity (9.8 m/s^2), measured in Newtons, N.

$\text{kg} \cdot \text{m/s}^2$

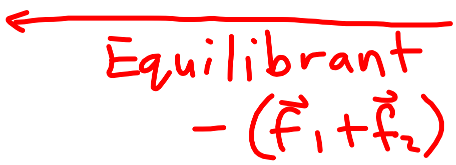
Minds On

Resultant and Composition of Forces

The resultant of several forces is the single force that can be used to represent the combined effect of all the forces. The individual forces that make up the resultant are called the components of the resultant.

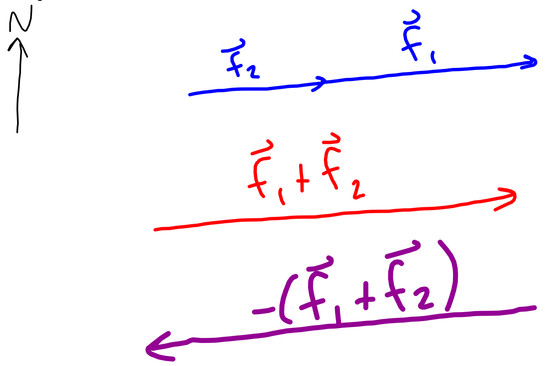
Equilibrant of Several Forces

The equilibrant is the opposite vector to the resultant. When the equilibrant is applied to the object, this force maintains the object in a state of equilibrium.



Minds On

Two children, James and Fred, are pushing on a rock. James pushes with a force of 80 N in an easterly direction, and Fred pushes with a force of 60 N in the same direction. Determine the resultant and the equilibrant of these two forces.



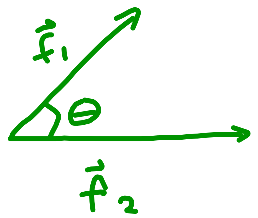
Newtons east

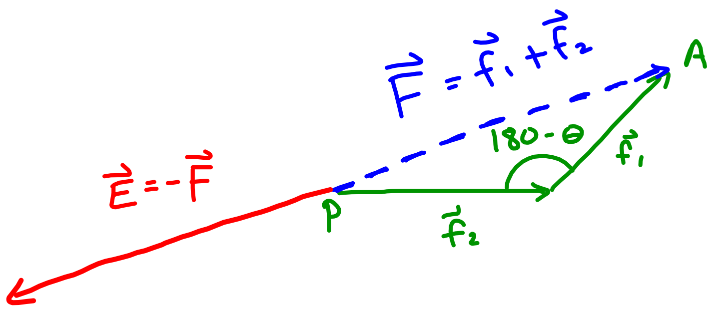
The resultant is 140N [E]

The equilibrant is 140N [W]

Action

Just like with vectors in Chapter 6, we can use a parallelogram or a triangle to determine the resultant and equilibrant vectors when two or more forces are combined, if they are not collinear. The notation is shown below:





Action

Forces in Equilibrium

When three noncollinear vectors are in a state of equilibrium, they will lie in the same plane and form a linear combination.

When arranged head to tail, they will form a triangle. (resultant of two of the forces is opposed by the third)

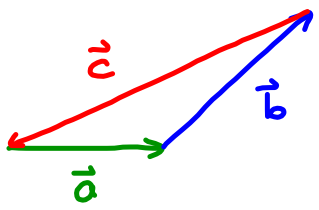
If three vectors ($\vec{a}$, $\vec{b}$, and $\vec{c}$) are in equilibrium, where $\vec{c}$ is the equilibriant of $\vec{a}$ and $\vec{b}$, then:

$$-\vec{c} = \vec{a} + \vec{b} \quad \textbf{OR} \quad \vec{a} + \vec{b} + \vec{c} = (-\vec{c}) + \vec{c} = \vec{0}$$

Forces in Equilibrium

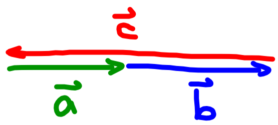
not parallel

Three non-collinear forces



$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

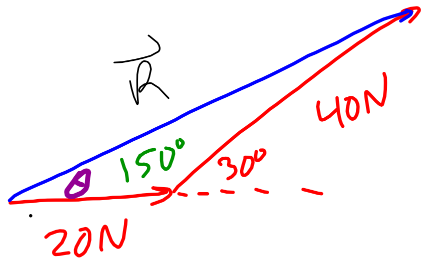
Three collinear forces



$\vec{a} + \vec{b} + \vec{c} = \vec{0}$

Action

Example 1: Two forces of 20 N and 40 N act at an angle of 30° to each other. Determine the resultant of these two forces.



$$|\vec{R}| = \sqrt{(20)^2 + (40)^2 - 2(20)(40)\cos 150^\circ}$$

$$\doteq 58.19\,N$$

$$\frac{\sin\theta}{40N} = \frac{\sin 150^\circ}{58.19N}$$

$$\theta = \sin^{-1}\left(\frac{40N \times \sin 150^\circ}{58.19N}\right)$$

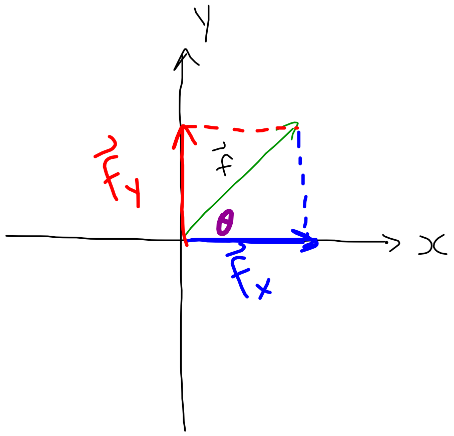
$$\theta \doteq 20.1^\circ$$

$\therefore$ the resultant is 58.19 N at an angle of 20.1°, counter-clockwise to the 20N force.

Action

Resolving a Vector into its Components

When we take a single force and break it into its two components, the process is called resolution. We can do this using the horizontal and vertical components of the force vector as shown:



$|\vec{f}_x| = |\vec{f}| \cos\theta$

$|\vec{f}_y| = |\vec{f}| \sin\theta$

$$\cos\theta = \frac{|\vec{f}_x|}{|\vec{f}|} \rightarrow |\vec{f}_x| = |\vec{f}|\cos\theta$$

Horizontal component

$$\sin\theta = \frac{|\vec{f}_y|}{|\vec{f}|} \rightarrow |\vec{f}_y| = |\vec{f}|\sin\theta$$

Vertical component

Action

Resolution of a Vector into Horizontal and Vertical Components

If the vector is resolved into its respective horizontal and vertical components, $\vec{f}_x$ and $\vec{f}_y$, then

$$\boxed{|\vec{f}_x| = |\vec{f}|\cos\theta}$$

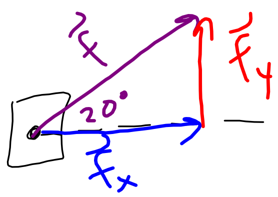
$$\boxed{|\vec{f}_y| = |\vec{f}|\sin\theta}$$

Action

Example 2: Kayla pulls on a rope attached to her sleigh with a force of 200 N. If the rope makes an angle of 20° with the horizontal, determine:

a) The force that pulls the sleigh forward

b) The force that tends to lift the sleigh

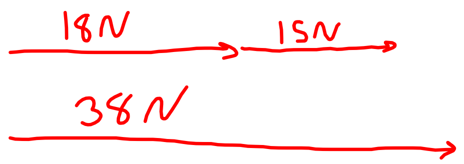


$|\vec{f}_x| = 200\text{N} \times \cos 20°$
$= 187.9\text{N}$

$|\vec{f}_y| = 200\text{N} \times \sin 20°$
$= 68.4\text{N}$

Action

Example 3: a) Is it possible for three forces of 15 N, 18 N, and 38 N to keep a system in a state of equilibrium? NO



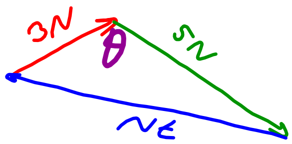
the sum of the magnitudes of the smaller two vectors must be greater than or equal to the magnitude of the longest vector.

b) Three forces having magnitudes 3 N, 5 N, and 7 N are in a state of equilibrium. Calculate the angle between the two smaller forces.

$$7^2 = 3^2 + 5^2 - 2(3)(5)\cos\theta$$

$$\theta = \cos^{-1}\left(\frac{7^2 - 3^2 - 5^2}{-2(3)(5)}\right)$$

$$= 120^\circ$$

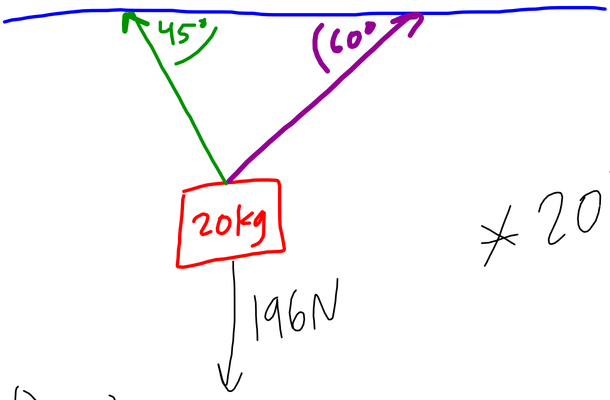


Action

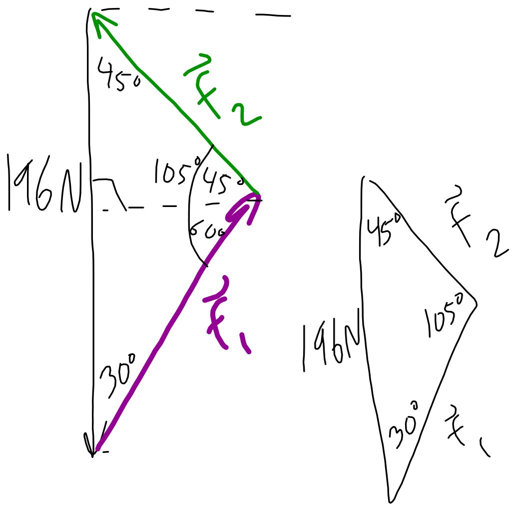
Example 4: A mass of 20 kg is suspended from a ceiling by two lengths of rope that make angles of 60° and 45° with the ceiling. Determine the tension in each of the ropes.

$\vec{f}_1$ $\vec{f}_2$

Method 1: Triangle of Forces



$* \; 20\,kg \times 9.8\,m/s^2$
$= 196\,kg \cdot m/s^2$
$= 196\,N$



$$\frac{|\vec{f}_1|}{\sin 45°} = \frac{|\vec{f}_2|}{\sin 30°} = \frac{196N}{\sin 105°}$$

$$|\vec{f}_1| = \frac{196N \times \sin 45°}{\sin 105°} \doteq 143.5N$$

$$|\vec{f}_2| = \frac{196N \times \sin 30°}{\sin 105°} \doteq 101.5N$$

Action

Example 4: A mass of 20 kg is suspended from a ceiling by two lengths of rope that make angles of 60^{o} and 45^{o} with the ceiling. Determine the tension in each of the ropes.

Method 2: Resolution of Vectors