Learning Goal: I will be able to use vectors to model and solve real-world problems involving velocity.

Minds On: The trouble with wind...groups

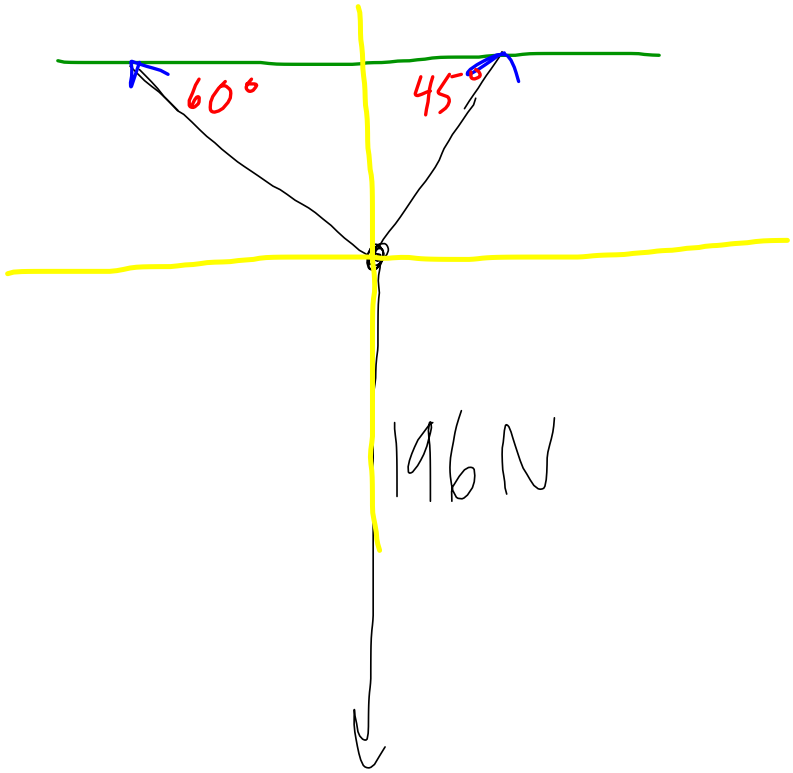
Action: 1. Example - boating!
2. Practice on page 369

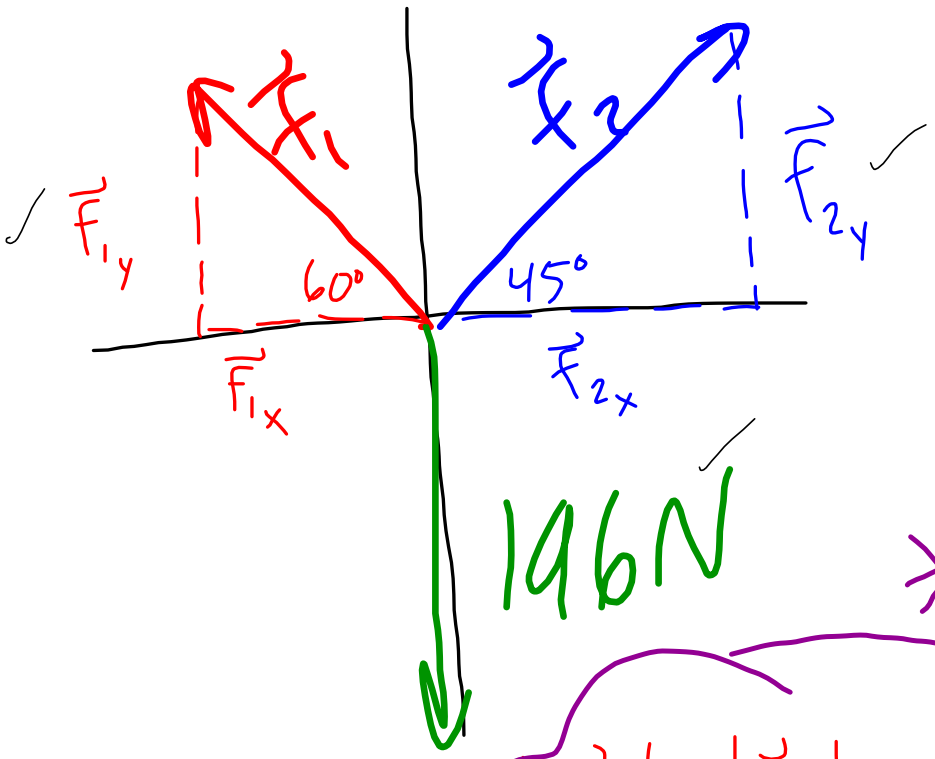
Consolidation: Exit Question

Minds On

Example 4: A mass of 20 kg is suspended from a ceiling by two lengths of rope that make angles of 60° and 45° with the ceiling. Determine the tension in each of the ropes.

Method 2: Resolution of Vectors





$|\vec{F}_{1y}| + |\vec{F}_{2y}| = 196\ N$

$|\vec{F}_{1x}| = |\vec{F}_{2x}|$

$|\vec{F}_{1x}| = |\vec{F}_1| \cos 60°$

$|\vec{F}_{1y}| = |\vec{F}_1| \sin 60°$

$|\vec{F}_{2x}| = |\vec{F}_2| \cos 45°$

$|\vec{F}_{2y}| = |\vec{F}_2| \sin 45°$

$\therefore$

① $|\vec{F}_1|\sin 60° + |\vec{F}_2|\sin 45° = 196\text{N}$

② $|\vec{F}_1|\cos 60° = |\vec{F}_2|\cos 45°$

① $0.8660|\vec{F}_1| + 0.7071|\vec{F}_2| = 196\text{N}$

② $0.5|\vec{F}_1| = 0.7071|\vec{F}_2|$

Rearrange ②

③ $|\vec{F}_1| = 1.4142|\vec{F}_2|$

Sub ③ into ①

$0.8660(1.4142|\vec{F}_2|) + 0.7071|\vec{F}_2| = 196$

$1.2247|\vec{F}_2| + 0.7071|\vec{F}_2| = 196$

$1.9319|\vec{F}_2| = 196\text{N}$

$|\vec{F}_2| = \frac{196\text{N}}{1.9319}$

④ $|\vec{F}_2| = 101.46\text{N}$

Sub ④ into ③

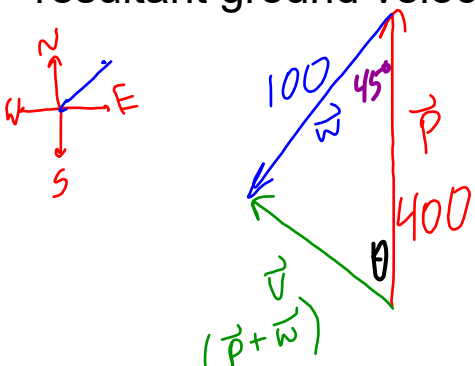
$|\vec{F}_1| = 1.4142(101.46\text{N})$

$= 143.48\text{N}$

$\therefore$ tension in rope 1 is 143.48N, and in rope 2 is 101.46N.

Minds On

A plane is heading due north with an air speed of 400 km/h when it is blown off course by a wind of 100 km/h from the northeast. Determine the resultant ground velocity of the airplane.



$$|\vec{v}|^2 = 100^2 + 400^2 - 2(100)(400)\cos 45°$$

$$|\vec{v}|^2 = 113{,}431.46$$

$$|\vec{v}| = 336.8 \text{ km/h}$$

speed ✓
direction?

$$\frac{\sin\theta}{100 \text{ km/h}} = \frac{\sin 45°}{336.8 \text{ km/h}}$$

$$\theta = \sin^{-1}\left(\frac{100 \sin 45°}{336.8}\right)$$

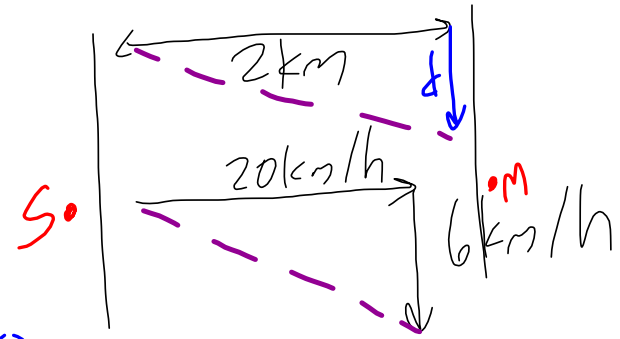
$$\theta = 12.1°$$

$\therefore$ flying 336.8 km/h [N 12.1° W]

Action

Example 1: A river is 2 km wide and flows at 6 km/h. Anna is driving a motorboat, which has a speed of 20 km/h in still water and she heads out from one bank in a direction perpendicular to the current. A marina lies directly across the river from the starting point on the opposite bank.

a) How far downstream from the marina will the current push the boat?

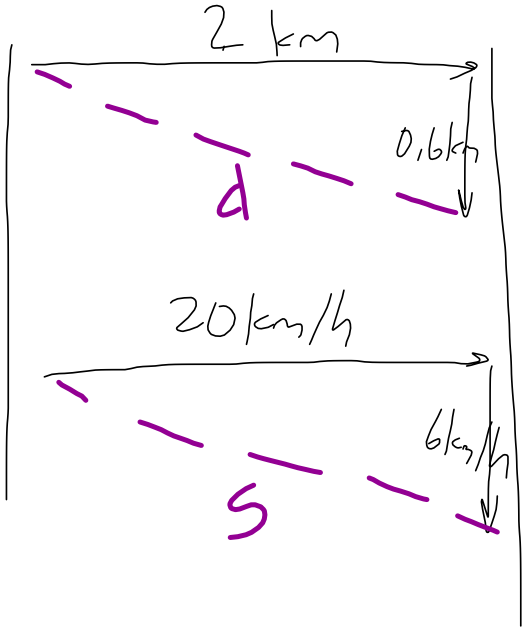


$$\frac{2}{d} = \frac{20}{6} \quad \text{OR} \quad \frac{2}{20} = \frac{d}{6}$$

$$d = \frac{2 \times 6}{20} = \frac{12}{20} = \frac{6}{10} = 0.6\text{ km}$$

Action

b) How long will it take for the boat to cross the river?



$d = \sqrt{2^2 + 0.6^2}$

$d = 2.09 \text{ km}$

$s = \frac{d}{t}$

$s = \sqrt{20^2 + 6^2}$

$s = 20.9 \text{ km/h}$

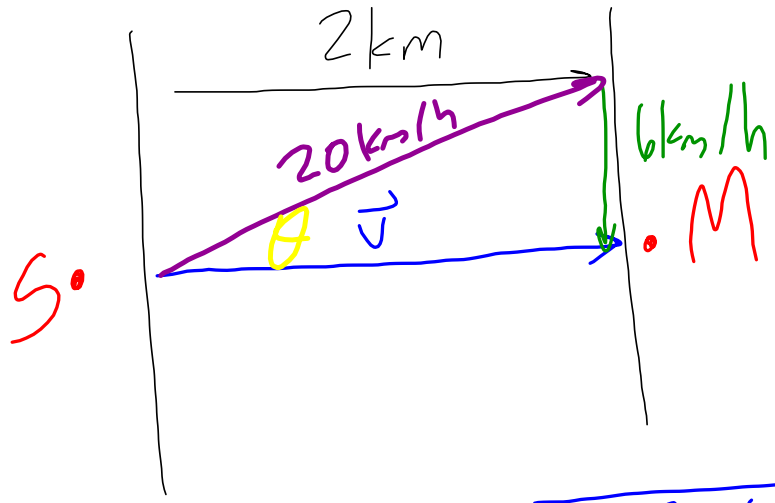
$t = \frac{d}{s} \quad t = \frac{2.09 \text{ km}}{20.9 \text{ km/h}}$

$= 0.1 \text{ h}$

$0.1 \text{h} \times 60 \text{m/h} = 6 \text{ minutes}$

Action

c) If Anna decides that she wants to end up directly across the river at the marina, in what direction should she head? What is the resultant velocity of the boat?



$$|\vec{v}| = \sqrt{20^2 - 6^2}$$
$$= \sqrt{364}$$
$$= 19.08$$

$$\sin\theta = \frac{6}{20}$$
$$\theta = 17.5^\circ$$

If she drives the boat at an angle of 17.5° upstream at 20km/h she will have a resultant velocity of 19.08km/h perpendicular to the current.

Consolidation

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