Learning Goal: I will be able to perform a dot product on two vectors.

Minds On: What do you think...?

Action: 1. Note
2. Practice on page 377

Consolidation: Exit Question

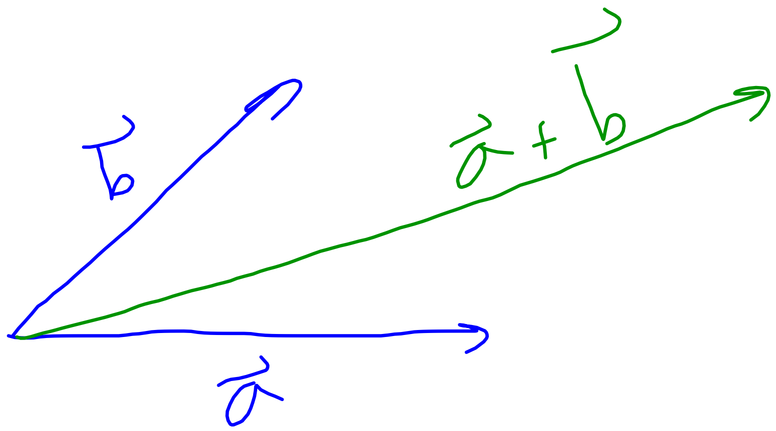
Minds On

Multiplying Vectors

You should be pretty familiar with adding and subtracting vectors, but what about multiplying??

What does multiplying vectors mean to you?

Brainstorm using word, pictures, etc.



Minds On

cos θ

What do you know about cos θ for various values of θ?

Think of possible angle values in a triangle. Let's not go above 180°.

4 $\cos \underline{90^\circ} = 0$

1 $\cos \underline{0^\circ} = 1$

2 $\cos \underline{45^\circ} = 0.7071$

7 $\cos 180^\circ = -1$

5 $\cos 120^\circ = -0.5$

3 $\cos 60^\circ = 0.5$

6 $\cos 135 = -0.7071$

$\sin 90^\circ = 1$

$\sin 0^\circ = 0$

$\sin 45^\circ = 0.7071$

$\sin 120^\circ = 0.8660$

$\sin 60^\circ = 0.8660$

$\sin 135^\circ = 0.7071$

Action

7.3 Dot Product

The dot product of two vectors deals specifically with geometric vectors (vectors that do not have a coordinate system associated with them). The dot product between two geometric vectors $\vec{a}$ and $\vec{b}$ is a **scalar quantity** defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|cos\theta$, where θ is the angle between the two vectors. The vectors must be placed tail-to-tail when calculating the dot-product. The dot product is only calculated for vectors when the angle between the vectors is between 0° and 180°.

Action

- If θ < 90°, then $\vec{a} \cdot \vec{b} > 0$
- If θ = 90°, then $\vec{a} \cdot \vec{b} = 0$
- If 90° < θ ≤ 180°, then $\vec{a} \cdot \vec{b} < 0$

based on cos θ

- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (commutative property)
- $\vec{a} \cdot (\vec{b} + \vec{c}) = (\vec{a} \cdot \vec{b}) + (\vec{a} \cdot \vec{c})$ (distributive property)

$|\vec{a}||\vec{b}| \cos\theta$
$+ |\vec{a}||\vec{c}| \cos\alpha$

- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ (magnitudes property)
- $\vec{i} \cdot \vec{i} = 1, \vec{j} \cdot \vec{j} = 1$ and $\vec{k} \cdot \vec{k} = 1$
- $(k\vec{a}) \cdot \vec{b} = \vec{a} \cdot (k\vec{b}) = k(\vec{a} \cdot \vec{b})$ (associative property with a scalar)

Action

Example 1: Two vectors, $\vec{a}$ and $\vec{b}$, are placed tail to tail and have magnitudes 3 and 5, respectively. There is an angle of 120° between the vectors. Calculate $\vec{a} \cdot \vec{b}$.

$$\begin{aligned} \vec{a} \cdot \vec{b} &= |\vec{a}||\vec{b}| \cos\theta \\ &= (3)(5)\cos 120^\circ \\ &= 15 \cos 120^\circ \\ &= -7.5 \end{aligned}$$

Action

Example 2: a) If $|\vec{a}| = \sqrt{7}$, calculate $\vec{a} \cdot \vec{a}$.

b) Calculate $\vec{i} \cdot \vec{i}$.

a) $\vec{a} \cdot \vec{a} = |\vec{a}||\vec{a}| \cos\theta$

$= (\sqrt{7})(\sqrt{7}) \cos 0^\circ$

$= 7(1)$

$= 7$

b) $\vec{i} \cdot \vec{i} = |\vec{i}||\vec{i}| \cos\theta$

$= (1)(1) \cos 0^\circ$

$= 1$

Action

Example 3: If the vectors $\vec{a}+3\vec{b}$ and $4\vec{a}-\vec{b}$ are perpendicular, and $|\vec{a}| = 2|\vec{b}|$, determine the angle (to the nearest degree) between the nonzero vectors $\vec{a}$ and $\vec{b}$.

$$(\vec{a}+3\vec{b})\cdot(4\vec{a}-\vec{b})=0$$

$$0=(\vec{a}\cdot 4\vec{a})-(\vec{a}\cdot\vec{b})+12(\vec{a}\cdot\vec{b})-(3\vec{b}\cdot\vec{b})$$

$$0=4|\vec{a}|^2-(\vec{a}\cdot\vec{b})+12(\vec{a}\cdot\vec{b})-3|\vec{b}|^2$$

$$0=4|\vec{a}|^2+11(\vec{a}\cdot\vec{b})-3|\vec{b}|^2$$

$$0=4|\vec{a}|^2+11(|\vec{a}||\vec{b}|\cos\theta)-3|\vec{b}|^2$$

$$0=4(2|\vec{b}|)^2+11((2|\vec{b}|)|\vec{b}|\cos\theta)-3|\vec{b}|^2$$

$$0=16|\vec{b}|^2+11(2|\vec{b}|^2\cos\theta)-3|\vec{b}|^2$$

$$0=13|\vec{b}|^2+22|\vec{b}|^2\cos\theta$$

$$-13\cancel{|\vec{b}|^2}=22\cancel{|\vec{b}|^2}\cos\theta$$

$$\cos\theta=\frac{-13}{22}$$

$$\theta \doteq 126.2°$$

Action

Example 4: If $|\vec{x}+\vec{y}| = |\vec{x}-\vec{y}|$, prove that the nonzero vectors, $\vec{x}$ and $\vec{y}$, are perpendicular.

prove $\vec{x}\cdot\vec{y}=0$

$$\cancel{\vec{x}\cdot\vec{y}=0}$$
$$\cancel{|\vec{x}||\vec{y}|\cos\theta=0}$$
$$\cancel{|\vec{x}||\vec{y}|\cos 90^\circ=0}$$
$$\cancel{|\vec{x}||\vec{y}|(0)=0}$$
$$\cancel{0=0}$$

$$|\vec{a}|^2=\vec{a}\cdot\vec{a}$$

$$|\vec{x}+\vec{y}|^2=|\vec{x}-\vec{y}|^2$$

$$\cancel{|\vec{x}+\vec{y}|-|\vec{x}-\vec{y}|=0}$$

$$\cancel{|\vec{x}+\vec{y}||\vec{x}+\vec{y}|=|\vec{x}-\vec{y}||\vec{x}-\vec{y}|}$$

$$(\vec{x}+\vec{y})\cdot(\vec{x}+\vec{y})=(\vec{x}-\vec{y})\cdot(\vec{x}-\vec{y})$$

$$(\vec{x}\cdot\vec{x})+(\vec{x}\cdot\vec{y})+(\vec{y}\cdot\vec{x})+(\vec{y}\cdot\vec{y})=(\vec{x}\cdot\vec{x})-(\vec{x}\cdot\vec{y})-(\vec{y}\cdot\vec{x})+(\vec{y}\cdot\vec{y})$$

$$\cancel{|\vec{x}|^2}+2(\vec{x}\cdot\vec{y})+\cancel{|\vec{y}|^2}=\cancel{|\vec{x}|^2}-2(\vec{x}\cdot\vec{y})+\cancel{|\vec{y}|^2}$$

$$\frac{4(\vec{x}\cdot\vec{y})}{4}=\frac{0}{4}$$

$$\vec{x}\cdot\vec{y}=0$$

QED

$$(j + d)(c + e)$$