# 7.3 Dot Product

The dot product of two vectors deals specifically with geometric vectors (vectors that do not have a coordinate system associated with them). The dot product between two geometric vectors $\vec{a}$ and $\vec{b}$ is a **scalar quantity** defined as $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|cos\theta$, where θ is the angle between the two vectors. The vectors must be placed tail-to-tail when calculating the dot-product. The dot product is only calculated for vectors when the angle between the vectors is between 0° and 180°.

- **If θ < 90°, then $\vec{a} \cdot \vec{b} > 0$**
- **If θ = 90°, then $\vec{a} \cdot \vec{b} = 0$**
- **If 90° < θ ≤ 180°, then $\vec{a} \cdot \vec{b} < 0$**
- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ (commutative property)
- $\vec{a} \cdot (\vec{b} + \vec{c}) = (\vec{a} \cdot \vec{b}) + (\vec{a} \cdot \vec{c})$ (distributive property)
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$ (magnitudes property)
- $\vec{i} \cdot \vec{i} = 1, \vec{j} \cdot \vec{j} = 1 \; and \; \vec{k} \cdot \vec{k} = 1$
- $(k\vec{a}) \cdot \vec{b} = \vec{a} \cdot (k\vec{b}) = k(\vec{a} \cdot \vec{b})$ (associative property with a scalar)

***Example 1***: Two vectors, $\vec{a}$ and $\vec{b}$, are placed tail to tail and have magnitudes 3 and 5, respectively. There is an angle of 120° between the vectors. Calculate $\vec{a} \cdot \vec{b}$.

***Example 2***:

a) If $|\vec{a}| = \sqrt{7}$, calculate $\vec{a} \cdot \vec{a}$.

b) Calculate $\vec{i} \cdot \vec{i}$.

***Example 3***: If the vectors $\vec{a}$ + 3$\vec{b}$ and 4$\vec{a}$ – $\vec{b}$ are perpendicular, and $|\vec{a}| = 2|\vec{b}|$, determine the angle (to the nearest degree) between the nonzero vectors $\vec{a}$ and $\vec{b}$.

***Example 4***: If $|\vec{x} + \vec{y}| = |\vec{x} - \vec{y}|$, prove that the nonzero vectors, $\vec{x}$ and $\vec{y}$, are perpendicular.