My computer was broken this day.

See handout for written out questions and note.

Solving Linear Systems by Elimination

$3x + 2y = 7$ ①

$-4x - 2y = 2$ ②

add ① + ② $\quad -1x + 0y = 9$

$-1x = 9$

$x = -9$ ③

sub ③ into ①

$3(-9) + 2y = 7$

$2y = 34$

$y = 17$

$\therefore (x, y) = (-9, 17)$

$$6x - 3y = 2 \quad (1)$$
$$6x + 7y = 8 \quad (2)$$
$$0x - 10y = -6 \quad (1) - (2)$$

$$y = \frac{6}{10}$$

$$y = \frac{3}{5}$$

$2x - 7y = 12$ ①

$4x + 3y = 6$ ②

① × 2

$4x - 14y = 24$ ③

$4x + 3y = 6$ ②

③ − ② $0x - 17y = 18$

$y = -\frac{18}{17}$

$$3x - 2y = 6 \quad (1)$$

$$5x + 7y = 35 \quad (2)$$

$(1) \times 5 \qquad 15x - 10y = 30 \quad (3)$

$(2) \times 3 \qquad 15x + 21y = 105 \quad (4)$

$(3) - (4) \qquad 0x - 31y = -75$

$$y = \frac{75}{31}$$

Dot Product of Algebraic Vectors

$\vec{a} = (a_1, a_2, a_3) \quad \vec{b} = (b_1, b_2, b_3)$

$\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2 + a_3 b_3$

$\vec{x} \cdot \vec{y} = |\vec{x}||\vec{y}| \cos\theta$

$= x_1 y_1 + x_2 y_2 + x_3 y_3$

$= \vec{y} \cdot \vec{x}$

Example 2

$\vec{a} = (-1, 2, 4)$
$\vec{b} = (3, 4, 3)$

$$\vec{a} \cdot \vec{b} = (-1, 2, 4) \cdot (3, 4, 3)$$
$$= (-1)(3) + (2)(4) + (4)(3)$$
$$= -3 + 8 + 12$$
$$\boxed{\vec{a} \cdot \vec{b} = 17}$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{\cancel{|\vec{a}||\vec{b}|} \cos\theta}{\cancel{|\vec{a}||\vec{b}|}}$$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

$$\boxed{\theta = \cos^{-1}\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)}$$

$$\theta = \cos^{-1}\left(\frac{17}{\sqrt{(-1)^2+(2)^2+(4)^2}\sqrt{(3)^2+(4)^2+(3)^2}}\right)$$

$$= \cos^{-1}\left(\frac{17}{\sqrt{21}\sqrt{34}}\right)$$

$$\theta \doteq 50.5^\circ$$

Example 1

Prove $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

$\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} = (a_1, a_2, a_3)\cdot(b_1, b_2, b_3) + (a_1, a_2, a_3)\cdot(c_1, c_2, c_3)$

$= a_1b_1 + a_2b_2 + a_3b_3 + a_1c_1 + a_2c_2 + a_3c_3$

$= a_1(b_1+c_1) + a_2(b_2+c_2) + a_3(b_3+c_3)$

$= \vec{a}\cdot(\vec{b}+\vec{c})$ Q.E.D.

$\vec{a} = (-1, 3, -4)$

$\vec{b} = (3, k, -2)$

If $\vec{a} \perp \vec{b}$, then... $\vec{a} \cdot \vec{b} = 0$

$(-1, 3, -4) \cdot (3, k, -2) = 0$

$(-1)(3) + (3)(k) + (-4)(-2) = 0$

$-3 + 3k + 8 = 0$

$3k = -5$

$k = \frac{-5}{3}$

$\vec{x} = (m, m, -3)$

$\vec{y} = (m, -3, 6)$

* If $\vec{x} \perp \vec{y}$, then $\vec{x} \cdot \vec{y} = 0$

$(m, m, -3) \cdot (m, -3, 6) = 0$

$m^2 - 3m - 18 = 0$

$(m+3)(m-6) = 0$

$\therefore m = -3, 6$

$(-3, -3, -3) \perp (-3, -3, 6)$

$(6, 6, -3) \perp (6, -3, 6)$

mystery vector $\vec{z} = (x, y, z)$

$\vec{a} \cdot \vec{z} = 0$ & $\vec{b} \cdot \vec{z} = 0$

$(1, 5, -1) \cdot (x, y, z) = 0$

$x + 5y - z = 0$ ①

$(-3, 1, 2) \cdot (x, y, z) = 0$

$-3x + y + 2z = 0$ ②

$x + 5y - z = 0$ ①

$-3x + y + 2z = 0$ ②

①×3 $3x + 15y - 3z = 0$ ③

$-3x + y + 2z = 0$ ②

③+② $0x + 16y - z = 0$

$\boxed{z = 16y}$ ④

sub 4 into 1

$x + 5y - 16y = 0$

$\boxed{x = 11y}$ 5

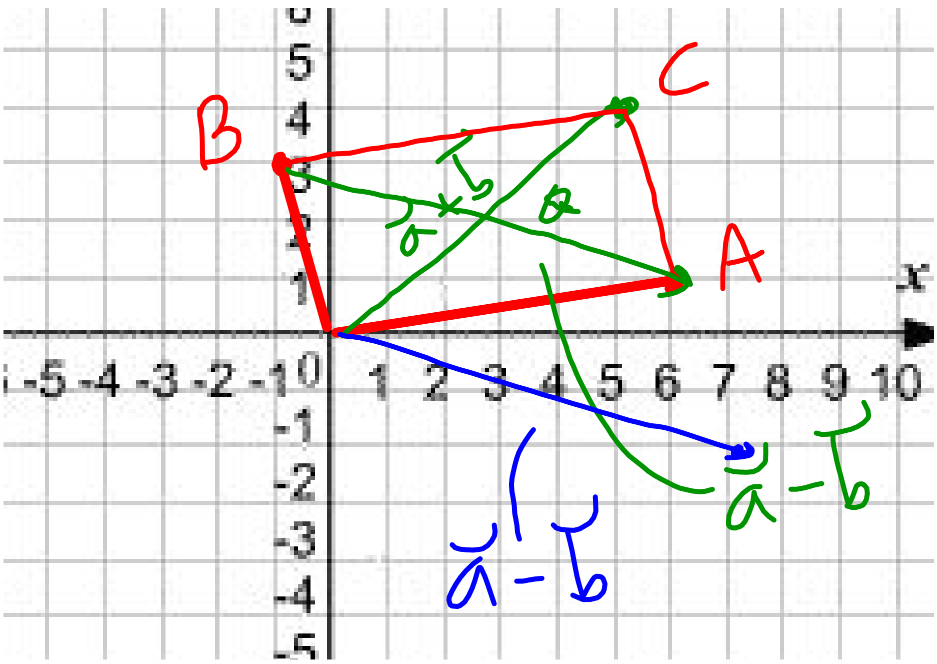
$\therefore$ vector $\vec{c} = (11y, y, 16y)$

is $\perp$ to both $\vec{a}$ and $\vec{b}$

$\therefore$ when $y = 1$

$\vec{c} = (11, 1, 16)$ is $\perp$

to both $\vec{a}$ and $\vec{b}$.



$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$$

<u>diagonals</u>

(a_1+b_1, a_2+b_2)

$\vec{a}+\vec{b} = (5, 4)$

(a_1-b_1, a_2-b_2)

$\vec{a}-\vec{b} = (7, -2)$

$$\cos\theta = \frac{(5,4)\cdot(7,-2)}{\sqrt{(5)^2+(4)^2}\sqrt{(7)^2+(-2)^2}}$$

$$= \frac{(5)(7)+(4)(-2)}{\sqrt{41}\sqrt{53}}$$

$$= \frac{27}{\sqrt{41}\sqrt{53}}$$

$$\cos\theta \doteq 0.5792$$

$$\theta = 54.6^\circ$$