7.5 - Scalar and Vector Projections

When two vectors, $\vec{a} = \overrightarrow{OA}$ and $\vec{b} = \overrightarrow{OB}$ are placed tail to tail, and θ is the angle between the vectors, $0^o \le \theta \le 180^o$, the scalar projection of $\vec{a}$ on $\vec{b}$ is ON, as shown below. The scalar projection can be determined using right triangle trigonometry and can be applied to either algebraic or geometric vectors equally well.

The scalar projection of $\vec{a}$ on $\vec{b}$ is obtained by drawing a line from the head of vector $\vec{a}$ perpendicular to $\overrightarrow{OB}$, or an extension of $\overrightarrow{OB}$.

Scalar Projection of $\vec{a}$ on $\vec{b}$

The scalar projection of vector $\vec{a}$ onto $\vec{b}$ is ON, where ON = $|\vec{a}|cos\theta$.

$0 \le \theta < 90^o$ $\qquad$ $90^o < \theta \le 180^o$ $\qquad$ $\theta = 90^o$

The scalar projection of vector $\vec{a}$ on vector $\vec{b}$ is in general not equal to the scalar projection of vector $\vec{b}$ on vector $\vec{a}$, which can be seen as follows:

Calculating the scalar projection of $\vec{a}$ on $\vec{b}$:

Calculating the scalar projection of $\vec{b}$ on $\vec{a}$:

Calculating Scalar Projections

The scalar projection of $\vec{a}$ on $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$. The scalar projection of $\vec{b}$ on $\vec{a}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$. In general, $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} \neq \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$.

Example 1: a) Show algebraically that the scalar projection of $\vec{a}$ on $\vec{b}$ is identical to the scalar projection of $\vec{a}$ on $2\vec{b}$.

b) Show algebraically that the scalar projection of $\vec{a}$ on $2\vec{b}$ is not the same as $\vec{a}$ on $-2\vec{b}$.

Example 2: For the vectors $\vec{a}$ = (-3, 4, 5√3) and $\vec{b}$ = (-2, 2, -1), calculate scalar projections of $\vec{a}$ on $\vec{b}$.

Example 3: Determine the angle that the vector $\overrightarrow{OP}$ = (2, 1, 4) makes with each of the coordinate axes.

Direction Cosines for $\overrightarrow{OP}$ = (a, b, c)

Example 4: For the vector $\overrightarrow{OP}$ = (-2√2, 4, -5), determine the direction cosine and corresponding angle that this vector makes with the positive z-axis.

Examining Vector Projections

The calculation of the vector projection of $\vec{a}$ on $\vec{b}$ is just the corresponding scalar projection of $\vec{a}$ on $\vec{b}$ multiplied by $\frac{\vec{b}}{|\vec{b}|}$. The expression $\frac{\vec{b}}{|\vec{b}|}$ is a unit vector pointing in the direction of $\vec{b}$.

Vector Projection of $\vec{a}$ on b

Vector projection of $\vec{a}$ on $\vec{b}$ = (scalar projection of $\vec{a}$ on $\vec{b}$)(unit vector in the direction of $\vec{b}$)

$= \left(\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}\right)\left(\frac{\vec{b}}{|\vec{b}|}\right)$ $\qquad = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\vec{b}$ $\qquad = \left(\frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\right)\vec{b}, \vec{b} \neq \vec{0}$

Example 5: Find the vector projection of $\overrightarrow{OA} = (4, 3)$ on $\overrightarrow{OB}$ = (4, -1).