Learning Goal: I will be able to solve problems involving the dot product and the cross product.

Minds On: Doorknobs

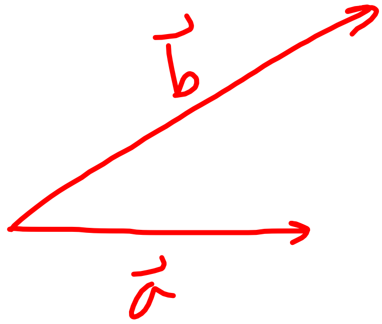
Action: Note and practice on page 414

Consolidation: Exit Question

Minds On

Common Mistakes from Test

Show $\vec{a} - \vec{b}$ (two ways / two different diagrams)



Write in unit vector form

$$(3,1,2) = 3\cancel{\vec{x}} + \cancel{\vec{y}} + 2\cancel{\vec{z}}$$

Angles in R^3?

Minds On

Cross Product

Determine the Cross Product

$\vec{a} = (3, -2, 1)$

$\vec{b} = (-5, 1, 6)$

$$\begin{array}{cc} a_2 & b_2 \\ a_3 & b_3 \\ a_1 & b_1 \\ a_2 & b_2 \end{array}$$

x: $a_2b_3 - a_3b_2$

y: $a_3b_1 - a_1b_3$

z: $a_1b_2 - a_2b_1$

$(-12-1, -5-18, 3-10)$

$(-13, -23, -7)$

Minds On

Read

Read Pg. 409

Action

7.7 Applications of the Dot Product and Cross Product

Physical *Application of the Dot Product*: When a force is acting on an object so that the object is moved from one point to another, we say that the force has done work. Work is defined as the product of the distance an object has been displaced and the component of the force along the line of displacement.

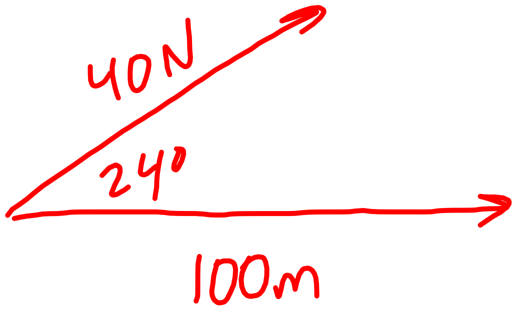
Formula for the Calculation of Work

W = $\vec{f} \cdot \vec{s}$, where $\vec{f}$ is the force acting on an object, measured in Newtons (N); $\vec{s}$ is the displacement of the object, measured in metres (m); and W is the work done, measured in joules (J).

$N \cdot m = J$

Action

Example 1: Marianna is pulling her daughter in a toboggan and is exerting a force of 40 N, acting at a 24° to the ground. If Marianna pulls the child a distance of 100 m, how much work was done?



$W = (|\vec{f}|\cos\theta)(|\vec{s}|)$

$W = \vec{f} \cdot \vec{s}$

$= |\vec{f}||\vec{s}|\cos\theta$

$= (40N)(100m)\cos 24°$

$= (4000 N\cdot m)(\cos 24°)$

$= 3{,}654.2$ J or $N\cdot m$

Action

Geometric Application of the Cross Product: The cross product of two vectors, $\vec{a}$ and $\vec{b}$, can be used to calculate the area of a parallelogram.

The formula for area of a parallelogram is $|\vec{a} \times \vec{b}|$, and $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|sin\theta$.

Example 2: a) Determine the area of the parallelogram determined by the vectors $\vec{p}$ = (-1, 5, 6) and $\vec{q}$ = (2, 3, -1).

b) Determine the area of the triangle formed by the points A(-1, 2, 1), B(-1, 0, 0), and C(3, -1, 4).

$$\text{area} = |\vec{p} \times \vec{q}| = |\vec{p}||\vec{q}|\sin\theta$$

$$= \left(\sqrt{(-1)^2+(5)^2+(6)^2}\right)\left(\sqrt{(2)^2+(3)^2+(-1)^2}\right)\sin\theta$$

$$\boxed{\begin{array}{cc} 5 & 3 \\ 6 & -1 \\ -1 & 2 \\ 5 & 3 \end{array}}$$

x, y, z

$$= |(-5-18,\ 12-1,\ -3-10)|$$

$$= |(-23, 11, -13)|$$

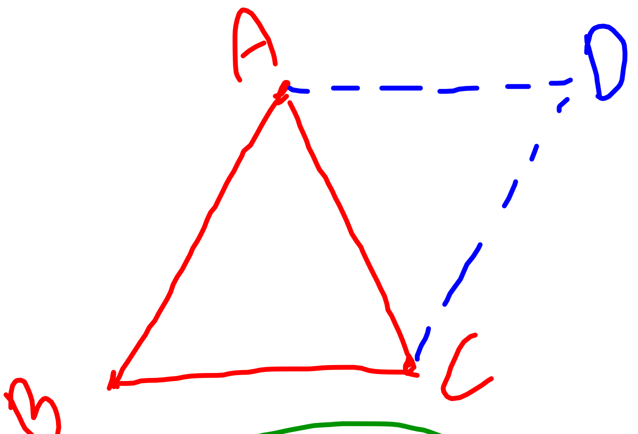
$$= \sqrt{(-23)^2+(11)^2+(-13)^2}$$

$$= \sqrt{819}$$

$$= 28.6 \text{ square units}$$

b) $A(-1,2,1)$
$B(-1,0,0)$
$C(3,-1,4)$

can't ~~do~~ find h :(



Find
$$\frac{\text{area}(ABCD)}{2}$$

÷ by 2!
elementary school style

$$\frac{|\vec{BC} \times \vec{BA}|}{2}$$

Find $\overrightarrow{BC}$ and $\overrightarrow{BA}$

$$\overrightarrow{BC} = (3,-1,4) - (-1,0,0)$$
$$= (4,-1,4)$$

$$\overrightarrow{BA} = (-1,2,1) - (-1,0,0)$$
$$= (0,2,1)$$

Find $|\overrightarrow{BC} \times \overrightarrow{BA}|$

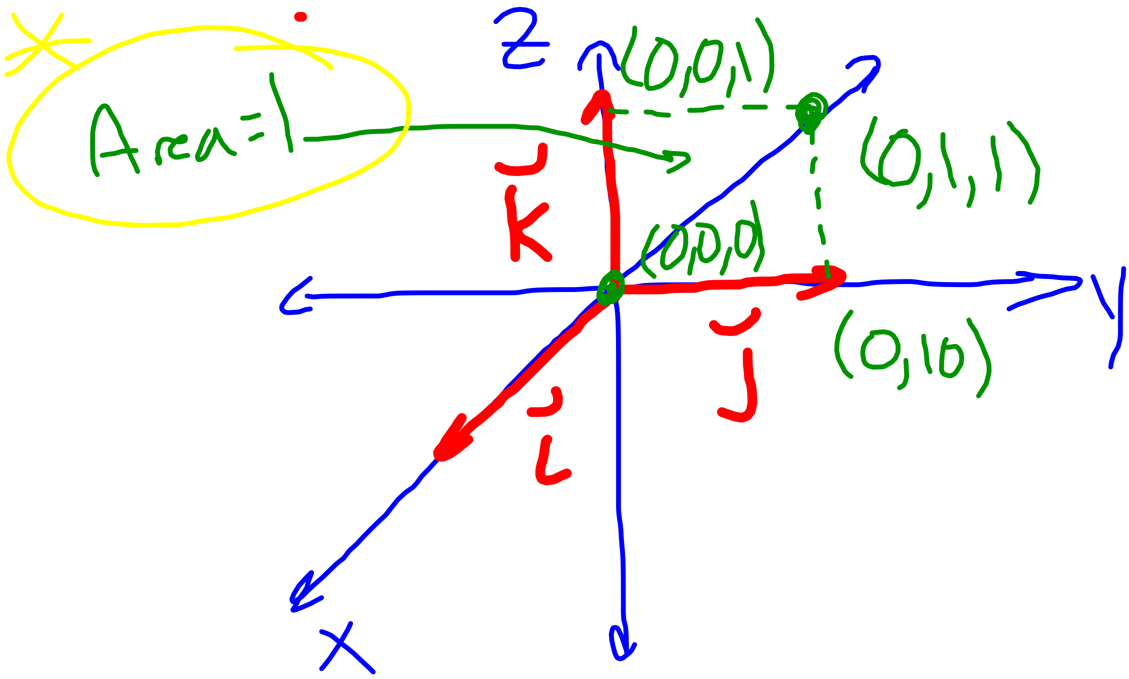
$$\begin{array}{cc} -1 & 2 \\ 4 & 1 \\ 4 & 0 \\ -1 & 2 \end{array}$$

$$|\overrightarrow{BC} \times \overrightarrow{BA}| = |(-1-8,\ 0-4,\ 8-0)|$$
$$= |(-9,-4,8)|$$
$$= \sqrt{81+16+64}$$
$$= \sqrt{161}$$

$$\text{Area of } \triangle ABC = \sqrt{161} \div 2$$
$$= \frac{1}{2}\sqrt{161}$$
$$\doteq \frac{1}{2}(12.7)$$
$$\doteq 6.3 \text{ square units}$$

Action

Example 3: Without calculating, explain why the cross product of $\vec{j}$ and $\vec{k}$ is $\vec{i}$.



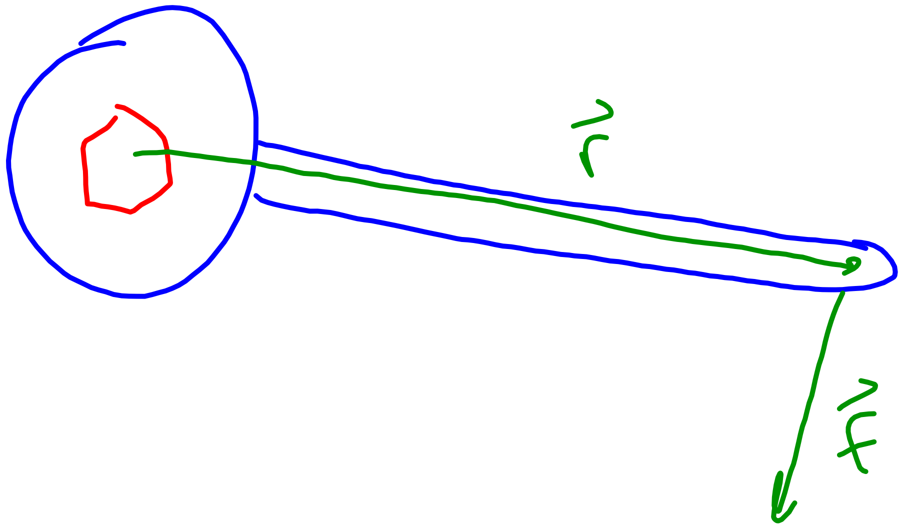
area of square = 1

$\therefore |\vec{j} \times \vec{k}| = 1$

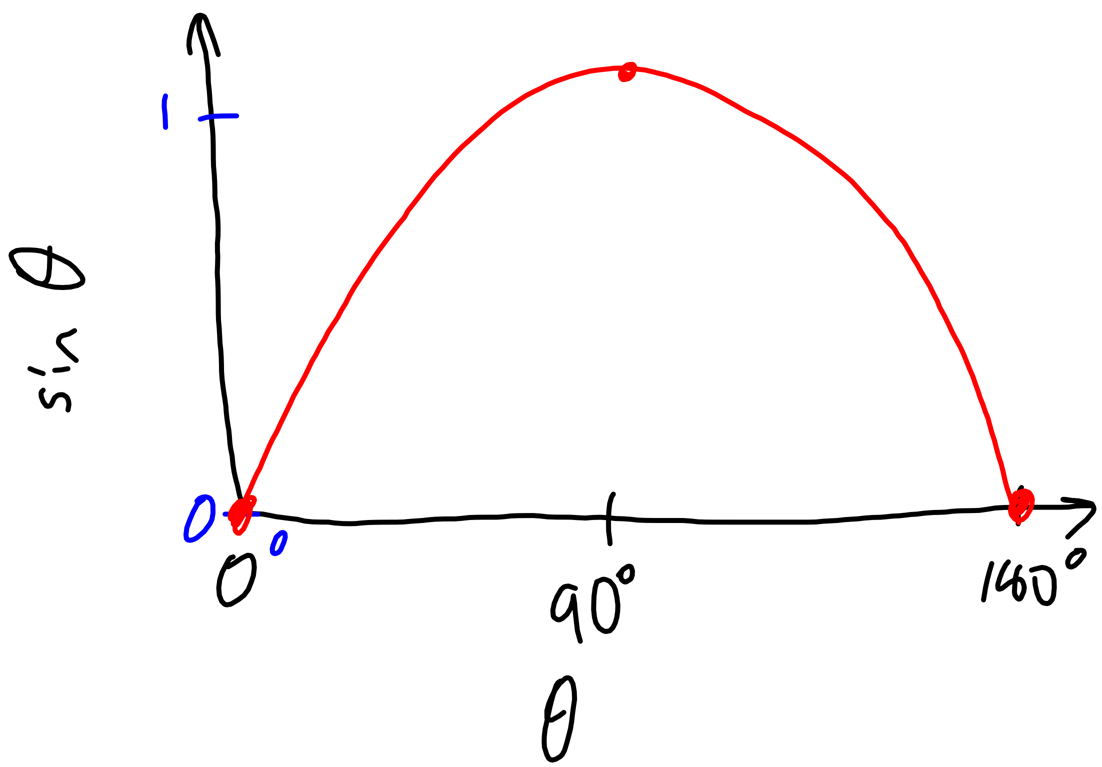
and $|\vec{j} \times \vec{k}|$ is $\perp$ to

$\vec{j}$ & $\vec{k}$

Read Pg. 413



Sketch sin x for 0<=x<=180

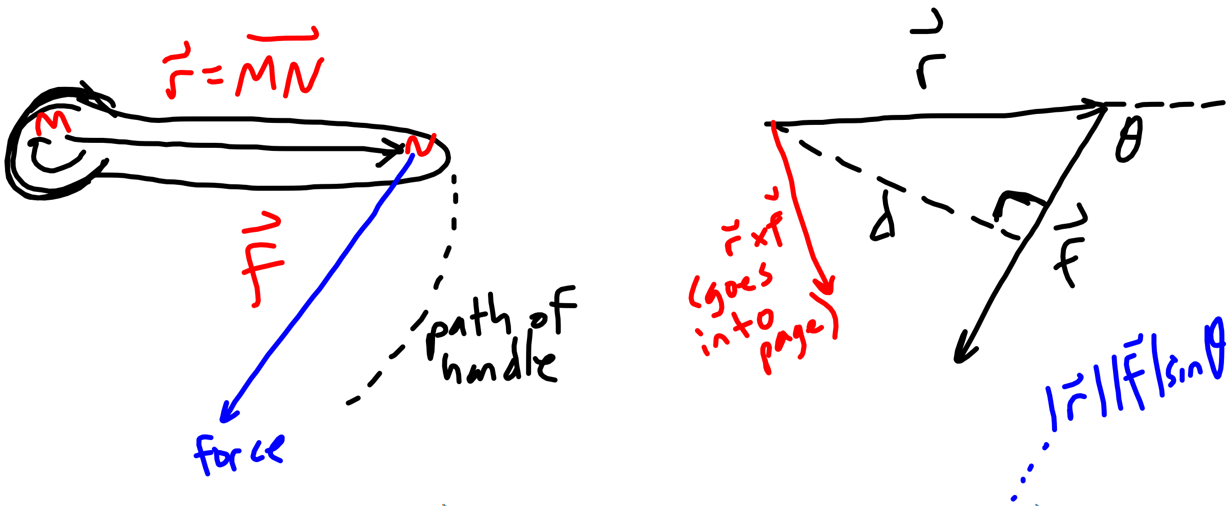


Action

Physical Application of the Cross Product: When we have forces that involve rotation, or turning about a point or an axis, we can use the cross product. This rotating force happens in everyday life when tightening or loosening a nut using a wrench, or when pedalling a bicycle, or when opening a door. In each of these examples, there is a rotation about either a point or an axis.

Action

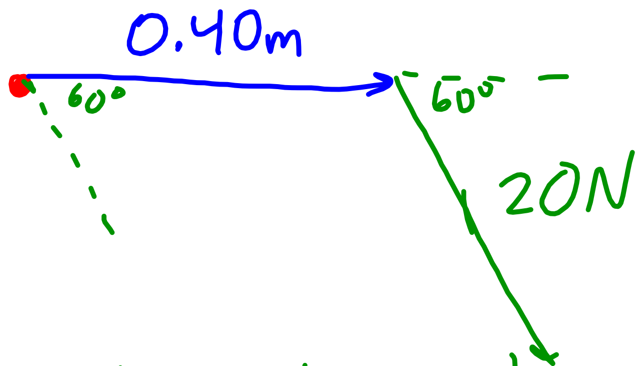
In the diagram below, a bolt with a right-hand thread is being screwed into a piece of wood by a wrench. The force, $\vec{f}$, is applied to the wrench at point N and is rotating about point M.



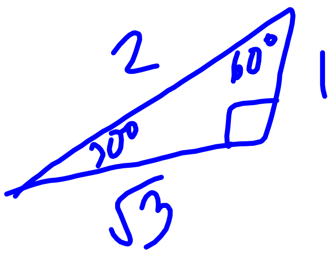
The torque, or the turning effect, of the force $\vec{f}$ about the point M is defined to be the vector $\vec{r} \times \vec{f}$. This vector is perpendicular to the vectors $\vec{r}$ and $\vec{f}$. To find its direction, we used a right-hand grip rule as follows: If you grip the imaginary axis of rotation at the point M so that your fingers point in the direction of the force, then your extended thumb points in the direction of the torque vector.

Action

Example 4: A 20 N force is applied at the end of a wrench that is 40 cm in length. The force is applied at an angle of 60º to the wrench. Calculate the magnitude of the torque about the point of rotation M.



$$|\text{Torque}| = |\vec{r} \times \vec{f}| = |\vec{r}||\vec{f}|\sin\theta$$



$$= (0.40\text{m})(20\text{N})\sin 60°$$

$$= 8\text{ N}\cdot\text{m} \times \frac{\sqrt{3}}{2}$$

$$= \frac{8\sqrt{3}}{2}\text{ J}$$

$$= 4\sqrt{3}\text{ J}$$

$$= 6.9\text{ J}$$

Consolidation

EXIT QUESTION

A wrench is used to tighten a bolt. A force of 60 N is applied in a clockwise direction at 80° to the handle, 20 cm from the centre of the bolt.

a) Calculate the magnitude of the torque
b) In what direction does the torque vector point?