Learning Goal: I will be able to make connections between Cartesian, vector and parametric equations of a line in R^2.

Minds On: Slope of a line formula

Action: Note

Consolidation: practice

A Few Concerns

The Vectors portion of this continues to build on itself. It is very important that you understand what we have done since the beginning of vectors.

Issues on Test

1. Definitions of dot product, cross product, scalar projection, vector projection, etc...
2. Two ropes hanging problem. (DIAGRAMS!)
3. Calculator errors
4. Reading the questions

Minds On

Whiteboards

Convert vector form equations into parametric form and vice versa.

$\vec{r} = (3,2) + t(2,-4)$

$x = 3 + 2t,\ y = 2 - 4t,\ t \in \mathbb{R}$

$x = -3t,\ y = 1 - t,\ t \in \mathbb{R}$

$\vec{r} = (0,1) + t(-3,-1)$

Minds On

Whiteboards

Are the two lines parallel?

Are they perpendicular?

$$\vec{r} = (3,2) + t(3,-2)$$

$$\vec{r} = (3,2) + t(12,8)$$

Minds On

Whiteboards

Create a line perpendicular to the given line.

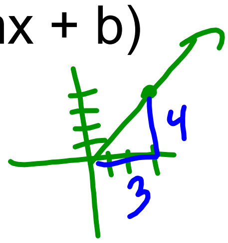
$$\vec{r} = (3,5) + t(2,-5)$$

$$\vec{r} = (3,5) + t(10,4)$$

Minds On

One More Whiteboard Question

Convert into slope y-intercept form. (y = mx + b)



$$\vec{r} = (1,2) + t(3,4)$$

slope

$$m = \frac{4}{3}$$

$$y = mx + b$$

$$b = y - mx$$

$$b = (2) - \left(\frac{4}{3}\right)(1)$$

$$b = 2 - \frac{4}{3}$$

$$b = \frac{6}{3} - \frac{4}{3}$$

$$b = \frac{2}{3}$$

$$y = \frac{4}{3}x + \frac{2}{3}$$

Action

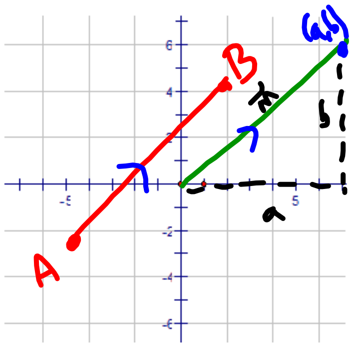
8.2 Cartesian Equation of a Line

Direction Vectors and Slope

In the diagram, a line segment AB with slope m = $\frac{b}{a}$ is shown with a run of a and a rise of b. The vector $\vec{m}=(a,b)$ is used to describe the direction of this line or any line parallel to it, with no restriction on the direction numbers a and b. In practice, a and b can be any two real numbers when describing a direction vector. If the direction vector of a line is m = (a, b), this corresponds to a slope of m = b/a except when a = 0. (why? can't ÷ by 0)

↓

vertical / undefined slope

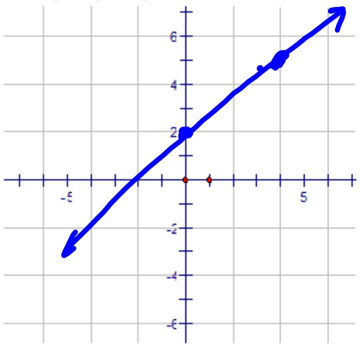


Action

xs rise xs run y-int

Example 1: Determine the equivalent vector and parametric equations of the line y = (3/4)x + 2.

$\vec{r} = (0,2) + t(4,3),\ t \in \mathbb{R}$



$x = 4t,\ y = 2 + 3t,\ t \in \mathbb{R}$

Action

run x rise y
x y a b

Example 2: For the line with equation $\vec{r}$ = (3, -6) + s(-1, -4), s ∈ R, determine the equivalent slope-y-intercept form.

$m = \frac{-4}{-1} = 4$

$y = mx + b$

$b = y - mx$

$b = (-6) - (4)(3)$

$= -6 - 12$

$= -14$

$\boxed{y = 4x - 14}$

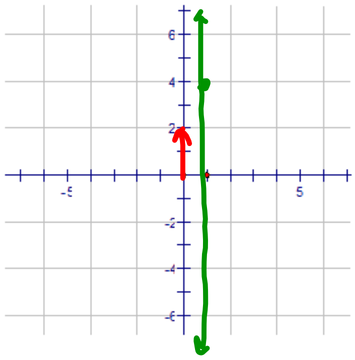
Action

Example 3: Determine the Cartesian form of the line with the equation $\vec{r} = (1, 4) + s(0, 2)$, $s \in R$.

$Ax + By + C = 0$

or

$Ax + By = C$



$x = 1$

$m = \frac{2}{0} = \underline{\text{undefined}}$

∴ equation is $x = \#$ where # is the x-value of any & every pt. on the line

Point given is $(1, 4)$ ∴ $x = 1$

$x - 1 = 0$

$Ax + By + C = 0$

Action

Cartesian Equation of a Line in R^2

In R^2, the Cartesian equation of a line (or scalar equation) is given by $Ax + By + C = 0$, where a normal to this line is $\vec{n} = (A, B)$.

A **normal** to this line is a vector drawn from the origin perpendicular to the given line to the point N(A, B).

Read Pg. 438 - 439

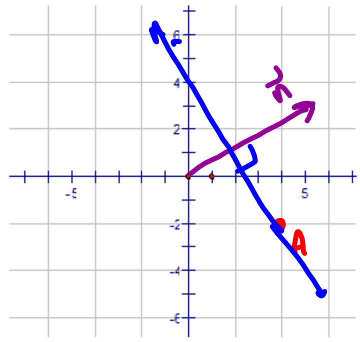
* remember $\vec{m} = (a, b)$ is direction vector

now $\vec{n} = (A, B)$ is normal

$\vec{m} \cdot \vec{n} = \boxed{0}$

Action

Example 4: Determine the Cartesian equation of the line passing through A(4, -2), which has $\vec{n}$= (5, 3) as its normal.



$Ax+By+C=0$

$A=5$

$B=3$

$5x+3y+C=0$

$5(4)+3(-2)+C=0$

$20-6+C=0$

$C=-14$

$\boxed{\therefore\ 5x+3y-14=0}$

Read Pg. 439 - 440 (Method 1)

Method 2 "Gilbert's Method"

$\vec{n} = (5, 3)$

$\vec{m} = (-3, 5)$ perpendicular (- reciprocal)

$m = \frac{-5}{3}$

$b = y - mx$

$b = (-2) - \left(\frac{-5}{3}\right)(4)$

$b = -2 + \frac{20}{3}$

$b = \frac{-6 + 20}{3}$

$b = \frac{14}{3}$

$$\therefore 3y = (3)\frac{-5}{3}x(3) + \frac{14}{3}$$

$$3y = \frac{-15}{3}x + \frac{42}{3}$$

$$3y = -5x + 14$$

$$\boxed{5x + 3y - 14 = 0}$$

Method 3

$$\vec{AP} = (x-4, y-(-2))$$
$$= (x-4, y+2)$$

This is line segment vector parallel to our line.

We know $\perp$, so $\vec{AP} \cdot \vec{n} = 0$

$$(x-4, y+2)\cdot(5,3) = 0$$

$$5(x-4) + 3(y+2) = 0$$

$$5x - 20 + 3y + 6 = 0$$

$$5x + 3y - 14 = 0$$

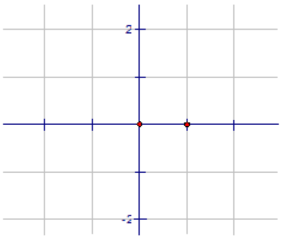
Action

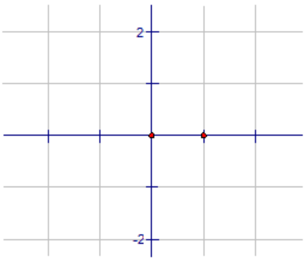
Parallel and Perpendicular Lines and their Normals

If the lines L_1 and L_2 have normals $\vec{n_1}$ and $\vec{n_2}$, respectively, we know the following:

1. The two lines are parallel IFF their normals are scalar multiples, $\vec{n_1} = k\vec{n_2}$, $k \in R$, $k \neq 0$. It follows that the direction vectors of the lines are also scalar multiples in this case.

2. The two lines are perpendicular IFF their dot product is zero, $\vec{n_1} \bullet \vec{n_2} = 0$. It follows that the dot product of the direction vectors is also zero in this case.





Action

Example 5: a) Show that the lines L_1: $3x - 4y - 6 = 0$ and L_2: $6x - 8y + 12 = 0$ are parallel and non-coincident.

b) For what value of k are the lines L_3: $kx + 4y - 4 = 0$ and L_4: $3x - 2y - 3 = 0$ perpendicular lines?

a) $\parallel$ because

$\vec{n}_1 = (3, -4)$ $\times 2$ $\quad \vec{n}_2 = 2\vec{n}_1$

$\vec{n}_2 = (6, -8)$

$\therefore \parallel$

If coincident $C_2 = 2C_1$

L.S.	R.S.
$= 12$	$= 2(-6)$
	$= -12$

$\therefore C_2 \neq 2C_1$

$\therefore$ lines are non-coincident

$$\vec{n}_1 \cdot \vec{n}_2 = 0$$

$$(k, 4) \cdot (3, -2) = 0$$

$$(k)(3) + (4)(-2) = 0$$

$$3k - 8 = 0$$

$$\boxed{k = +\frac{8}{3}}$$

Action

Example 6: Determine the acute angle formed at the point of intersection created by the following pair of lines:

L_1: (x,y) = (2,2) + s(-1,3), s ∈ R L_2: (x,y) = (5,1) + t(3,4), t ∈ R

$$\tilde{a} \cdot \tilde{b} = |\tilde{a}||\tilde{b}| \cos\theta$$

$$\cos\theta = \frac{\tilde{a} \cdot \tilde{b}}{|\tilde{a}||\tilde{b}|}$$

$\boxed{\text{angle btw } \tilde{m}_1 + \tilde{m}_2 = \text{angle btw } L_1 \text{ and } L_2}$

$$\therefore \cos\theta = \frac{\tilde{m}_1 \cdot \tilde{m}_2}{|\tilde{m}_1||\tilde{m}_2|}$$

$$= \frac{(-1,3)\cdot(3,4)}{\sqrt{(-1)^2+(3)^2}\sqrt{(3)^2+(4)^2}}$$

$$= \frac{(-1)(3)+(3)(4)}{(\sqrt{10})(\sqrt{25})}$$

$$= \frac{(-3)+12}{5\sqrt{10}}$$

$$= \frac{9}{5\sqrt{10}}$$

$$\cos\theta = 0.5692$$

$$\theta = 55.3^\circ$$