New Unit

Trigonometric Functions

Minds On

What's a Radian?

To this point we have measured angles in degrees.

Sometimes in math and physics we need a way to represent angles as pure numbers, without units.

In these situations, we use **radians**.

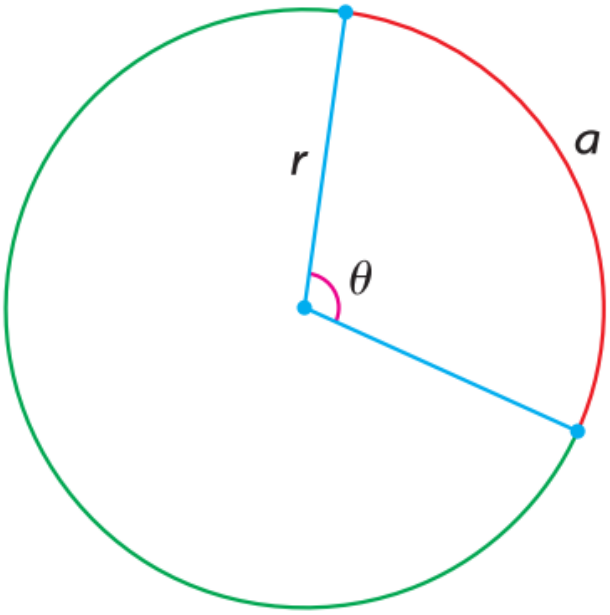
Minds On

What's a Radian?

When dealing in radians, the size of an angle is expressed in terms of the length of an arc, *a*, that subtends the angle θ, at the centre of a circle with radius *r*.

In this situation, *a* is proportional to *r* and to θ, where

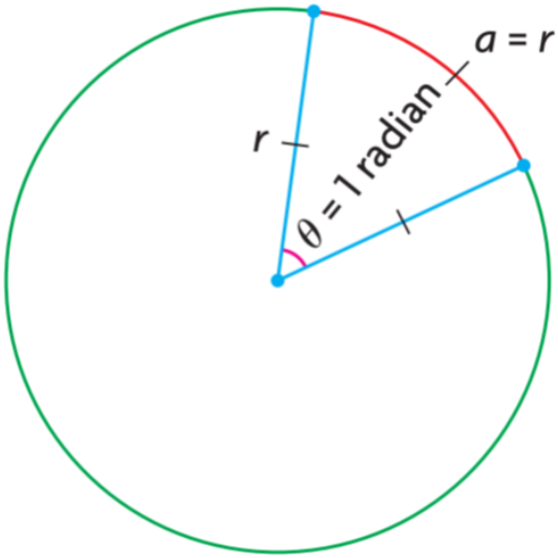
$$\theta = \frac{a}{r}$$



Action

How many degrees in 1 radian?

1 radian is defined as the angle subtended by an arc length, *a*, equal to the radius, *r*.

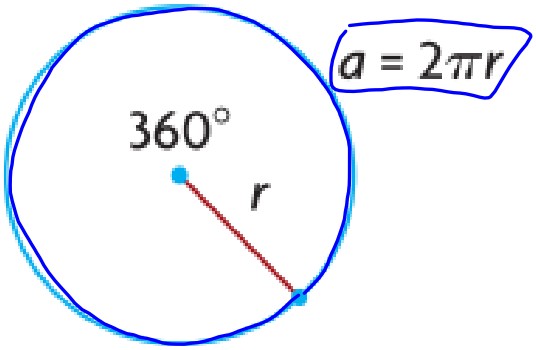


How many degrees are in 1 radian?

Action

How many degrees in 1 radian?

Let's start by considering the arc length created by an angle of 360°.



Remember that, in radians, $\theta = \frac{a}{r}$

$\theta = \frac{2\pi r}{r}$

360° = 2π radians

$360^\circ = 2\pi$ radians

$$\frac{360^\circ}{2} = \frac{2\pi}{2}$$

$$\frac{180^\circ}{\pi} = \frac{\pi \text{ radians}}{\pi}$$

$1 \text{ radian} = 57.3^\circ$

$\boxed{180^\circ = \pi \text{ radians}}$

Action

Converting from Degrees to Radians

Convert 90° to radians

$\frac{180°}{2} = \frac{\pi}{2}$ radians

$360° = 2\pi$ radians

$90° = \frac{\pi}{2}$ radians

$= 1.57$ radians

Action

Converting from Degrees to Radians

Convert 135° to radians

$180° = \pi$ radians

$\frac{180°}{\pi}$

$\frac{180°}{\pi} \quad \frac{135°}{x}$

$\frac{180°x}{180°} = \frac{135° \times \pi}{180°}$

$x = \frac{135\pi}{180}$

$x = \frac{27\pi}{36}$

$x = \frac{3\pi}{4}$

Action

Converting from Degrees to Radians

To convert from degrees to radians,

multiply by $\frac{\pi}{180^{o}}$

then reduce

Action

Converting from Degrees to Radians

Convert 30° to radians

$$30^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{30\pi}{180}$$

$$= \frac{\pi}{6}$$

Action

Converting from Degrees to Radians

Convert 225° to radians

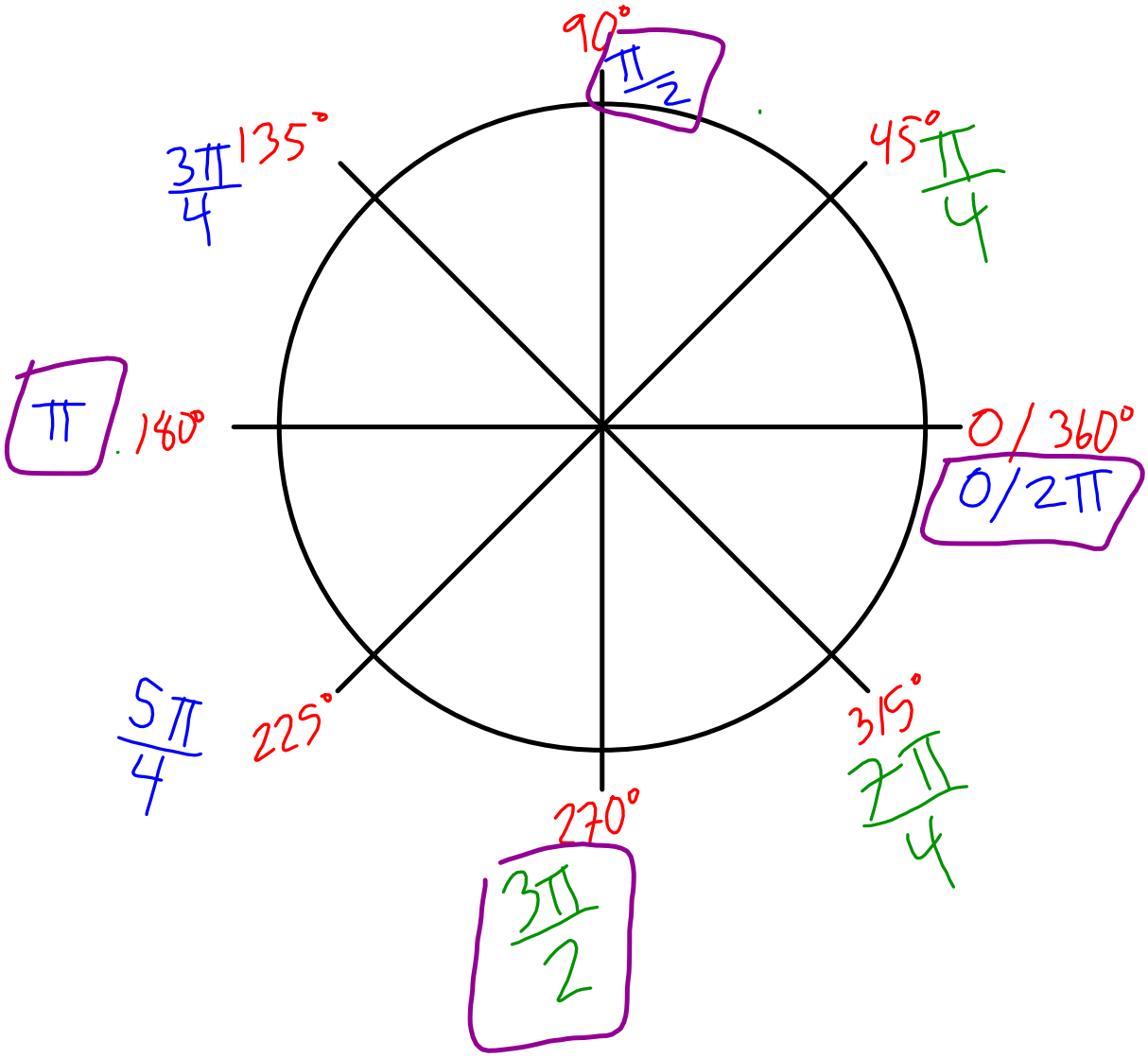
$$225^\circ \times \frac{\pi}{180^\circ}$$

$$= \frac{225\pi}{180}$$

$$= \frac{45\pi}{36}$$

$$= \frac{15\pi}{12}$$

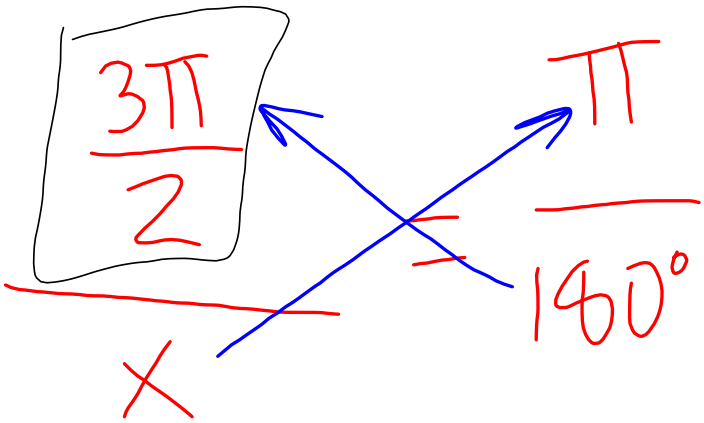
$$= \frac{5\pi}{4}$$


90°
π/2
45°
π/4
135°
3π/4
π
180°
0/360°
0/2π
225°
5π/4
315°
7π/4
270°
3π/2

Action

Converting from Radians to Degrees

Convert 3π/2 radians to degrees



$$\frac{3\pi}{2} \times 180^\circ = \pi \times x$$

$$x = \frac{3 \times 180}{2}$$

$$x = 270^\circ$$

Action

Converting from Radians to Degrees

To convert from radians to degrees,

multiply by $\frac{180^\circ}{\pi}$ **(or)** replace π with 180° (sometimes)

$\frac{3\pi}{2} \times \frac{180}{\pi} = 270^\circ$

Action

Converting from Radians to Degrees

Convert 5π/6 radians to degrees

$$\frac{5\pi}{6} \times \frac{180^\circ}{\pi}$$

$$= \frac{5 \times 180^\circ}{6}$$

$$= 150^\circ$$

Action

Converting from Radians to Degrees

Convert 1.75 radians to degrees

$$1.75 \times \frac{180^\circ}{\pi}$$

$$= 100.3^\circ$$

Action

Angular Velocity

Angular, or rotational, velocity is the amount of rotation a spinning object undergoes per unit time.

for example:

270° per second

or

3000 rotations per minute

or

$\frac{3\pi}{2}$ radians per second

Action

Angular Velocity

The London Eye ferris wheel has a diameter of 135 m and completes one revolution every 30 minutes.

Determine the angular velocity, ω, in radians per second.

$s = \frac{d}{t}$

convert 30 min. into s

$30 \text{ min} \times 60 \text{ s/min} = 1800 \text{ s}$

$$\omega = \frac{2\pi \text{ radians}}{1800 \text{ s}}$$

$$\omega = \frac{\pi}{900} \text{ radians/s}$$

$$\omega = 0.0035 \text{ radians/s}$$

Action

Angular Velocity

The London Eye ferris wheel has a diameter of 135 m and completes one revolution every 30 minutes.

How far has a rider travelled 10 minutes into the ride?

$\frac{1}{3}$ of a rotation $\left(\frac{10}{30}\right)$

$$\text{distance travelled} = \frac{1}{3}(2\pi r)$$

$$= \frac{2\pi(67.5)}{3}$$

$$= 141.4 \text{ m}$$

Consolidation

Working with Radians

A wheel is rotating at an angular velocity of 1.2π radians per second, while a point on the circumference of the wheel travels at 9.6π metres in 10 seconds.

How many revolutions does the wheel make in 1 minute? (60 seconds)

* 1 revolution is 2π radians

1.2π radians/sec × 60 sec/min

72π radians per minute

2π radians per rotation

36 rotations per minute

Consolidation

Working with Radians

A wheel is rotating at an angular velocity of 1.2π radians per second, while a point on the circumference of the wheel travels at 9.6π metres every 10 seconds.

What is the radius of the wheel?

$\theta = \frac{a}{r}$ (θ: angle, a: arc length, r: radius)

$1.2\pi = \frac{0.96\pi}{r}$ (per second)

$r = \frac{0.96\pi}{1.2\pi}$

$r = 0.8\text{m}$

Pg. 320

1, 5 - 8, 11, 14