Minds On

What effect do the parametersa, k, c and d have on a graph?

a - vertical stretch, compression, reflection
$|a|>1$ $0<|a|<1$ negative

k - horizontal, stretch, compression, reflection
$0<|k|<1$ $|k|>1$ negative

c - vertical translation

d - horizontal translation

Minds On

Which coordinate does each parameter apply to?

a y (multiply)

k x (divide)

c y (add)

d x (add)

Action

Example 1: Sketch these transformations that are applied to the graph of y = sin x, where 0 ≤ x ≤ 2π.

- A vertical stretch by a factor of 3
- A horizontal compression by a factor of ½
- A horizontal translation $\frac{\pi}{6}$ to the left
- A vertical translation 1 down

$$y = a\sin(k(x-d)) + c$$
$$y = 3\sin\left(2\left(x+\frac{\pi}{6}\right)\right) - 1$$

x	y
0	0
π/2	1
π	0
$\frac{3\pi}{2}$	−1
2π	0

⟹

$\frac{x}{2} - \frac{\pi}{6}$	y × 3 − 1
$-\frac{\pi}{6}$	−1
π/12	2
π/3	−1
7π/12	−4
5π/6	−1

$$0 \rightarrow \frac{0}{2} - \frac{\pi}{6}$$

$$= 0 - \frac{\pi}{6}$$

$$= -\frac{\pi}{6}$$

$$\frac{\pi}{2} \rightarrow \frac{\frac{\pi}{2}}{\frac{2}{1}} - \frac{\pi}{6}$$

$$= \left(\frac{3}{3}\right)\frac{\pi}{4} - \frac{\pi}{6}\left(\frac{2}{2}\right)$$

$$= \frac{3\pi - 2\pi}{12}$$

$$= \frac{\pi}{12}$$

$$\pi \rightarrow \left(\frac{3}{3}\right)\frac{\pi}{2} - \frac{\pi}{6}$$

$$= \frac{3\pi}{6} - \frac{\pi}{6}$$

$$= \frac{2\pi}{6}$$

$$= \frac{\pi}{3}$$

$$\frac{3\pi}{2} \rightarrow \frac{\frac{3\pi}{2}}{2} - \frac{\pi}{6}$$

$$= \left(\frac{3}{3}\right)\frac{3\pi}{4} - \frac{\pi}{6}\left(\frac{2}{2}\right)$$

$$= \frac{9\pi}{12} - \frac{2\pi}{12}$$

$$= \frac{7\pi}{12}$$

$$2\pi \rightarrow \frac{2\pi}{2} - \frac{\pi}{6}$$

$$= \left(\frac{6}{6}\right)\pi - \frac{\pi}{6}$$

$$= \frac{6\pi}{6} - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

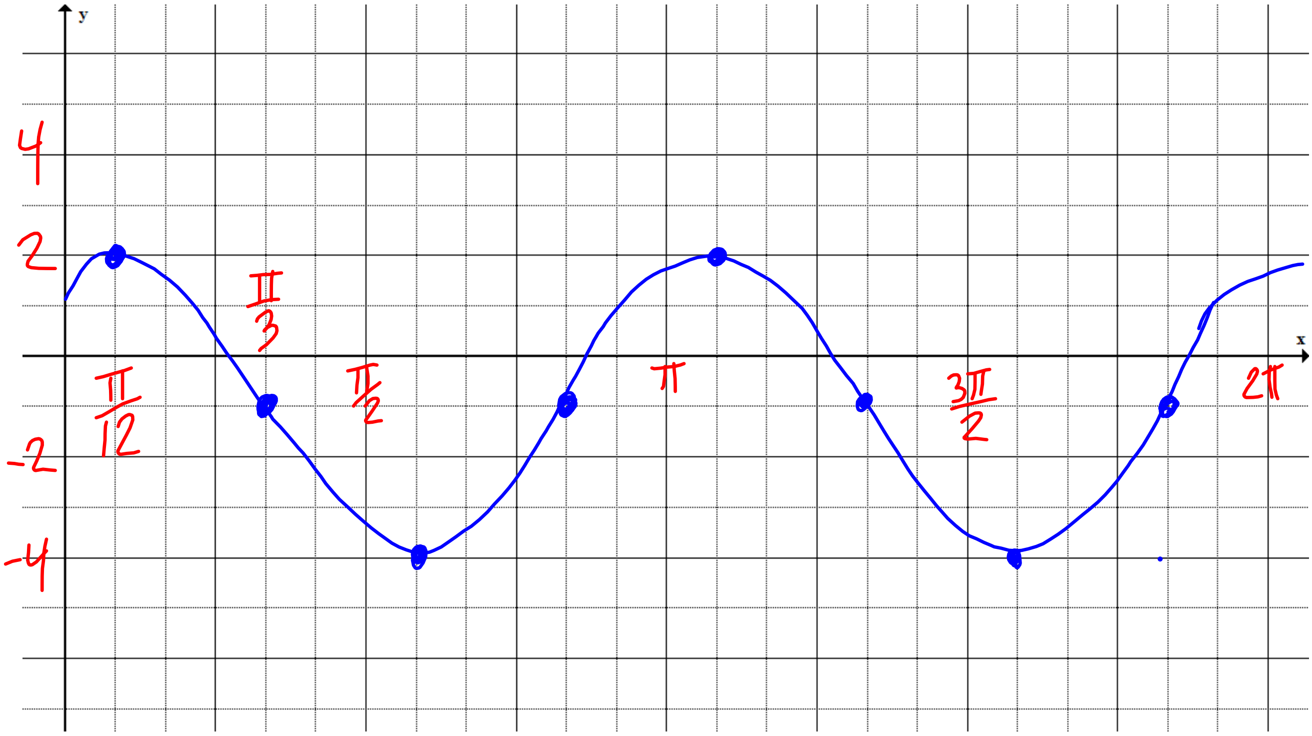
$\frac{x}{2} - \frac{\pi}{6}$	$y \times 3 - 1$
$-\pi/6$	-1
$\pi/12$	2
$\pi/3$	-1
$7\pi/12$	-4
$5\pi/6$	-1

Action

Example 1: Sketch these transformations that are applied to the graph of y = sin x, where $0 \le x \le 2\pi$.

- A vertical stretch by a factor of 3
- A horizontal compression by a factor of ½
- A horizontal translation $\frac{\pi}{6}$ to the left
- A vertical translation 1 down

Solution A: Apply transformations to key points of the parent function

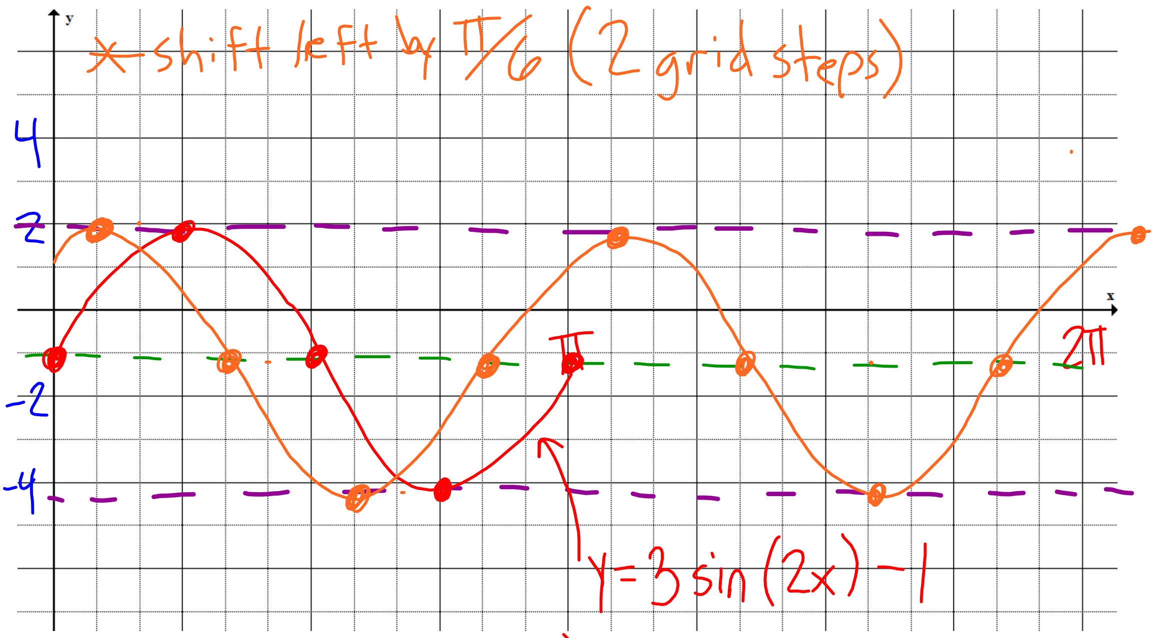


Action

Example 1: Sketch these transformations that are applied to the graph of $y = \sin x$, where $0 \le x \le 2\pi$.

- A vertical stretch by a factor of 3
- A horizontal compression by a factor of ½
- A horizontal translation $\frac{\pi}{6}$ to the left
- A vertical translation 1 down

Solution B: Use the features of the transformed function



$y = 3\sin(2(x+\pi/6)) - 1$

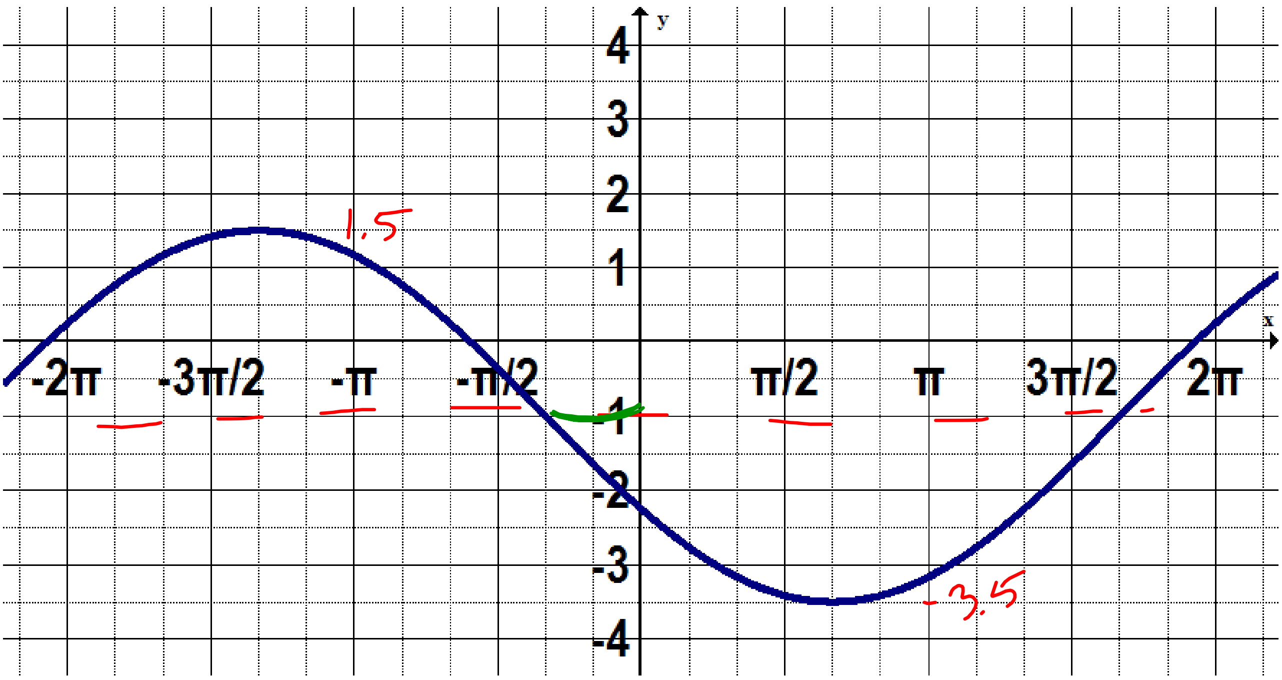
amplitude $= a = 3$ (distance from axis to max/min)

period $= \frac{2\pi}{k} = \pi$ ("time" for full cycle)

- equation of axis $= c = -1$ (center line through function)
- max $= c + a = -1 + 3 = 2$
- min $= c - a = -1 - 3 = -4$

Minds On

What's My Equation?



Solve on a whiteboard, show your process

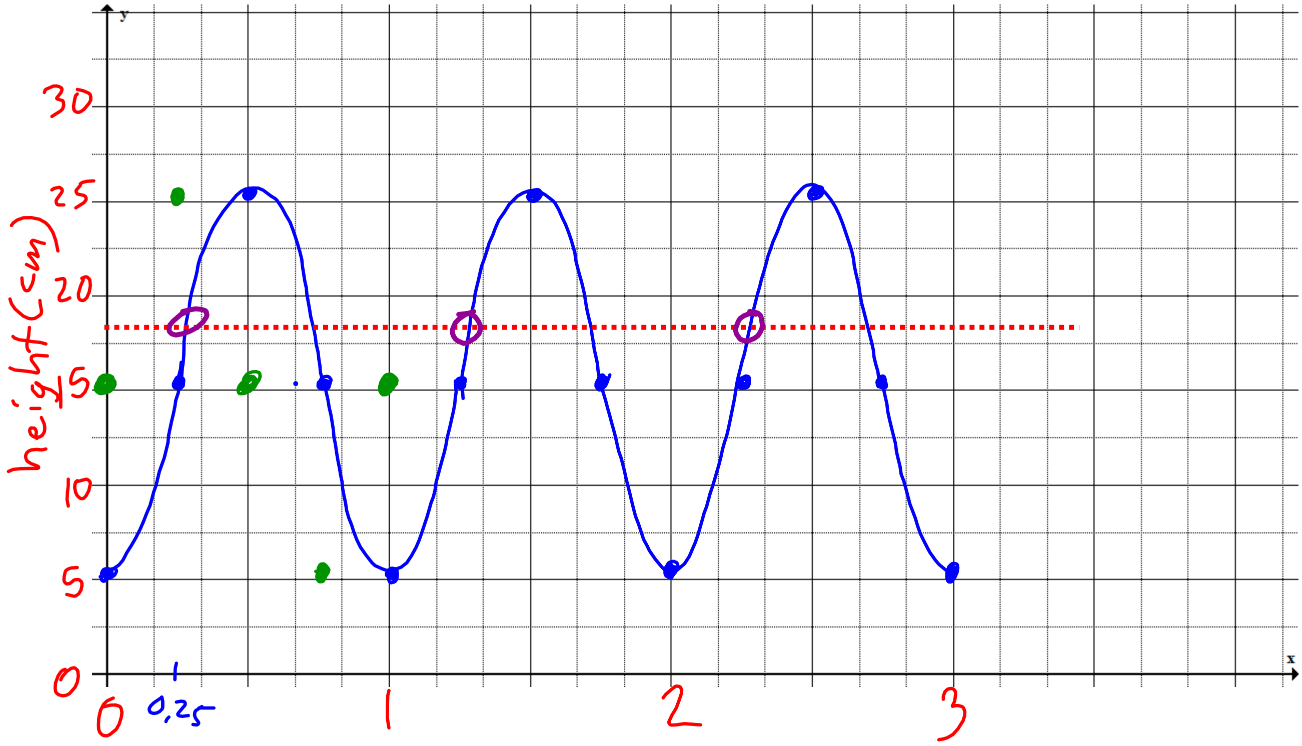
Action

Example 2: A mass on a spring is pulled toward the floor and released, causing it to move up and down. Its height, in centimetres, above the floor after t seconds is given by the function $h(t) = 10\sin(2\pi t + 1.5\pi) + 15, where\ 0 \le t \le 3$. Sketch a graph of height versus time. Then use your graph to predict when the mass will be 18 cm above the floor as it travels in an ~~upright~~ upward direction.

Factor out 2π

$h(t) = 10\sin(2\pi(t + 0.75)) + 15$

$\frac{1.5\pi}{2\pi} = 0.75$



axis: h=15 amplitude: 10 max: 25 min: 5

period: $\frac{2\pi}{k} = \frac{2\pi}{2\pi} = 1$

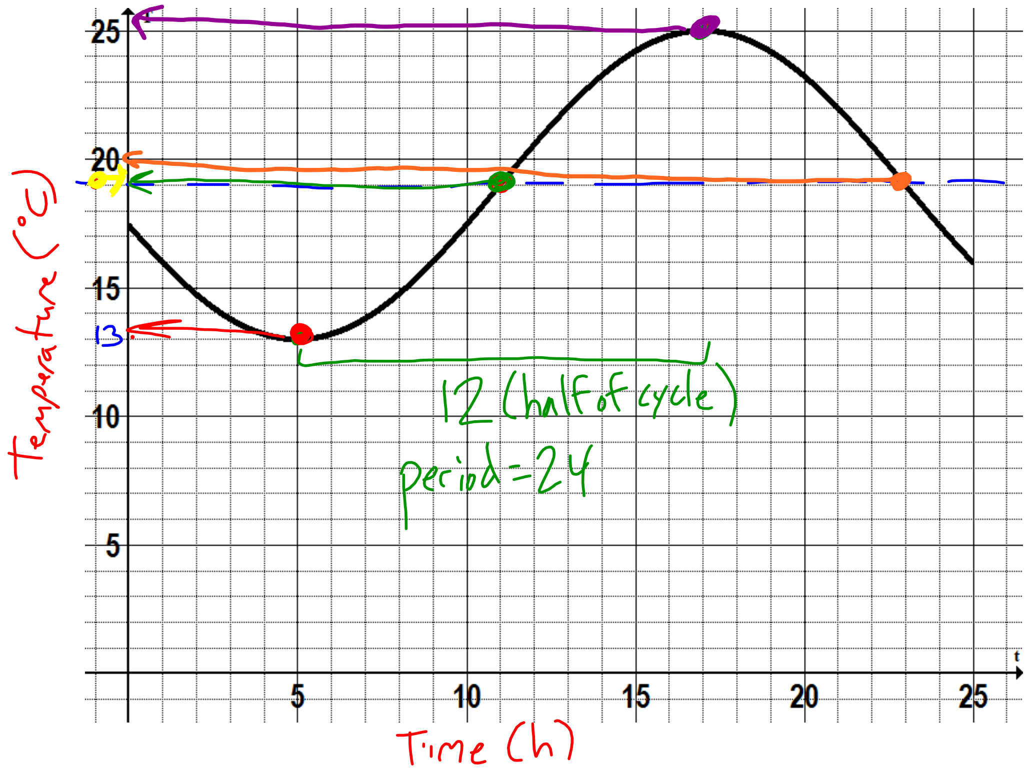
*we can shift 0.75 to the left OR 0.25 to the right

At a height of 18cm, travelling upwards, after 0.25s, 1.25s and 2.25s.

Action

Example 3: The following graph shows the temperature in Nellie's dorm room over a 24 h period.

Determine the equation of this sinusoidal function.



amplitude = 6
axis: T = 19
period: 24

$$\text{period} = \frac{2\pi}{k}$$

$$24 = \frac{2\pi}{k}$$

$$k = \frac{\pi}{12}$$

$$T(t) = -6\cos\left(\tfrac{\pi}{12}(t-5)\right) + 19$$

$$T(t) = 6\sin\left(\tfrac{\pi}{12}(t-11)\right) + 19$$

$$T(t) = 6\cos\left(\tfrac{\pi}{12}(t-17)\right) + 19$$

$$T(t) = -6\sin\left(\tfrac{\pi}{12}(t-23)\right) + 19$$

$$T(t) = -6\sin\left(\tfrac{\pi}{12}(t+1)\right) + 19$$

Action

Summary of Key Ideas

- The graphs of functions of the form f(x) = a sin(k(x – d)) + c and f(x) = a cos(k(x – d)) + c are transformations of the parent functions y = sin(x) and y = cos(x), respectively.
- The parameters a, k, d, and c give useful information about transformations and characteristics of the function.

Transformations of the Parent Function	Characteristics of the Transformed Function
$\lvert a \rvert$ gives the vertical stretch/compression factor. If a < 0, there is also a reflection in the x-axis.	$\lvert a \rvert$ gives the amplitude.
$\left\lvert \frac{1}{k} \right\rvert$ gives the horizontal stretch/compression factor. If k < 0, there is also a reflection in the y-axis.	$\left\lvert \frac{2\pi}{k} \right\rvert$ gives the period.
d gives the horizontal translation.	d gives the horizontal translation.
c gives the vertical translation.	y = c gives the equation of the axis.

- If the independent variable (x, t, etc) has a coefficient other than +1, the argument (angle) must be factored to separate the values of k and d. For example,

y = 3 cos(2x + π) should be changed to $y = 3\cos\left(2\left(x + \frac{\pi}{2}\right)\right)$.

yesterday

Pg. 344

1, 4, 5, 7

today

Pg. 344

8, 11, 14