Learning Goal: I will determine amplitude, vertical translation, phase shift, and period of a sinusoidal function given its equation.
Given a data set, I will model the data using a sinusoidal function.
Determine the equation of a sinusoidal function given its amplitude, vertical translation, phase shift, and period.

Minds On: Clickers - Transformations!

Action: Note - Modelling

Consolidation: Exit Question

Minds On

Determine the value of the function below at the given values of the independent variable.

$$f(x) = -2\sin\left(3\left(x - \frac{\pi}{2}\right)\right) + 4$$

6

4

2

f(0) = 2

f(1) = 5.98

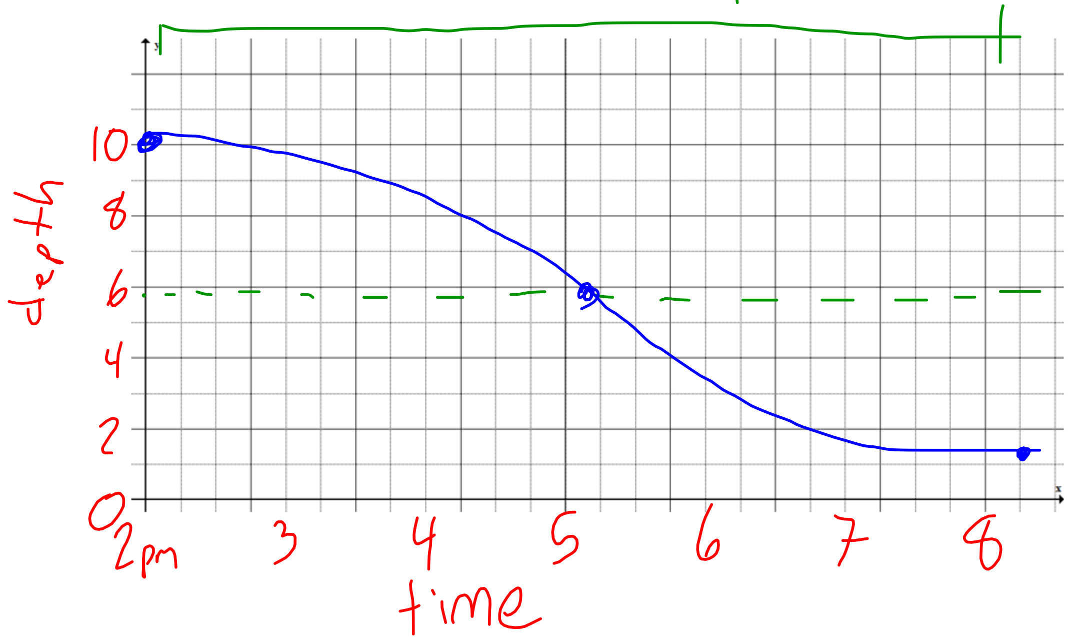
f(π) = 6

f(π/2) = 4

Modelling with Trigonometric Functions

Example 1: The tides at Cape Capstan, New Brunswick, change the depth of the water in the harbour. On one day in October, the tides have a high point of approximately 10 m at 2 p.m. and a low point of approximately 1.2 m at 8:15 p.m. A particular sailboat has a draft of 2 m. This means it can only move in water that is at least 2 m deep. The captain of this sailboat plans to exit the harbour at 6:30 p.m. Create a sinusoidal function to model the problem, and use it to determine whether the sailboat can exit the harbour safely at 6:30 p.m.

6.25 h



axis: $D = 5.6$

amplitude: 4.4

period: $6.25 \times 2 = 12.5$

$k = \frac{2\pi}{12.5} = \frac{4\pi}{25}$

$$D(t) = 4.4\cos\left(\frac{4\pi}{25}t\right) + 5.6$$

$6{:}30\text{pm} \rightarrow t = 6.5 - 2$
$= 4.5$

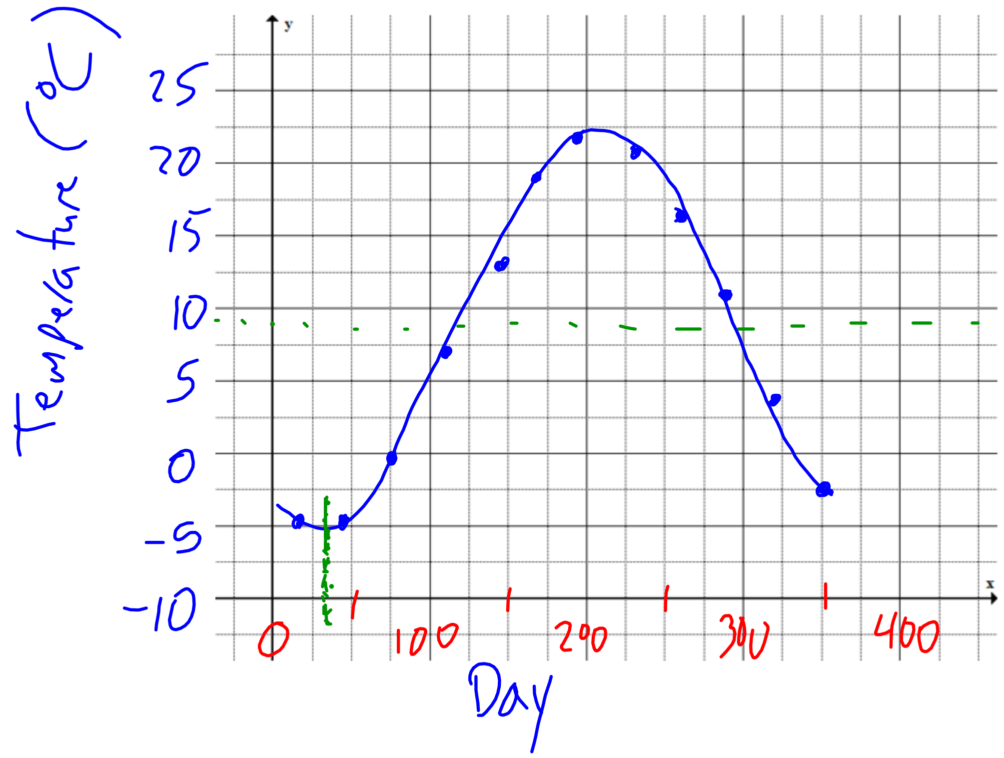
$D(4.5) = 2.6\text{m}$

$\therefore$ the boat is good to go!

Example 2: The following table shows the average monthly means of the daily (24 h) temperatures in Hamilton, Ontario. Each month's average temperature is represented by the day in the middle of the month.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Day	15	45	75	106	136	167	197	228	259	289	320	350
°C	-4.8	-4.8	-0.2	6.6	12.7	18.6	21.9	20.7	16.4	10.5	3.6	-2.3

a) Plot the temperature data and fit a sinusoidal curve to the points.
b) Estimate the average daily temperature in Hamilton on the 200th day of the year.



axis: $\frac{-5+22}{2} = 8.5$

Period: 365

$k = \frac{2\pi}{365}$

amplitude: 13.5

*use -cos

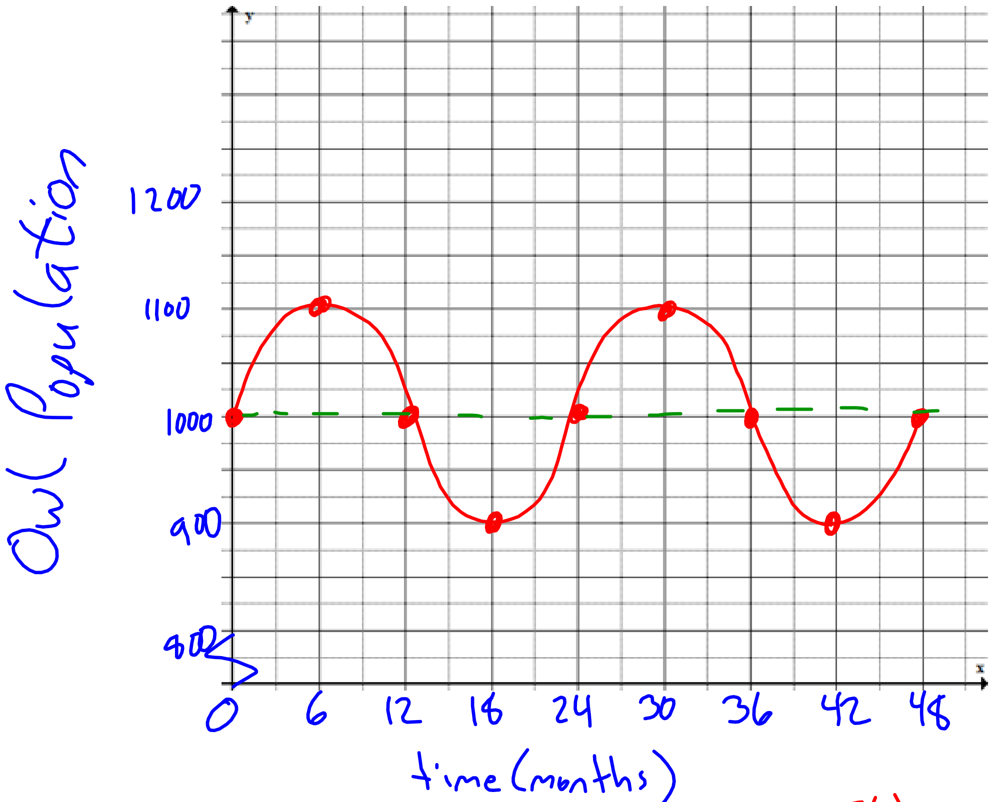
$T(t) = -13.5\cos\left(\frac{2\pi}{365}(t-30)\right) + 8.5$

$T(200) = 21.7°C$

<u>Example 3:</u> The population size, O, of owls (predators) in a certain region can be modelled by the function $\boxed{O(t) = 1000 + 100\sin(\frac{\pi t}{12})}$, where t represents the time in months and t = 0 represents January. The population size, m, of mice (prey) in the same region is given by the function $\boxed{m(t) = 20\,000 + 4000\cos(\frac{\pi t}{12})}$.

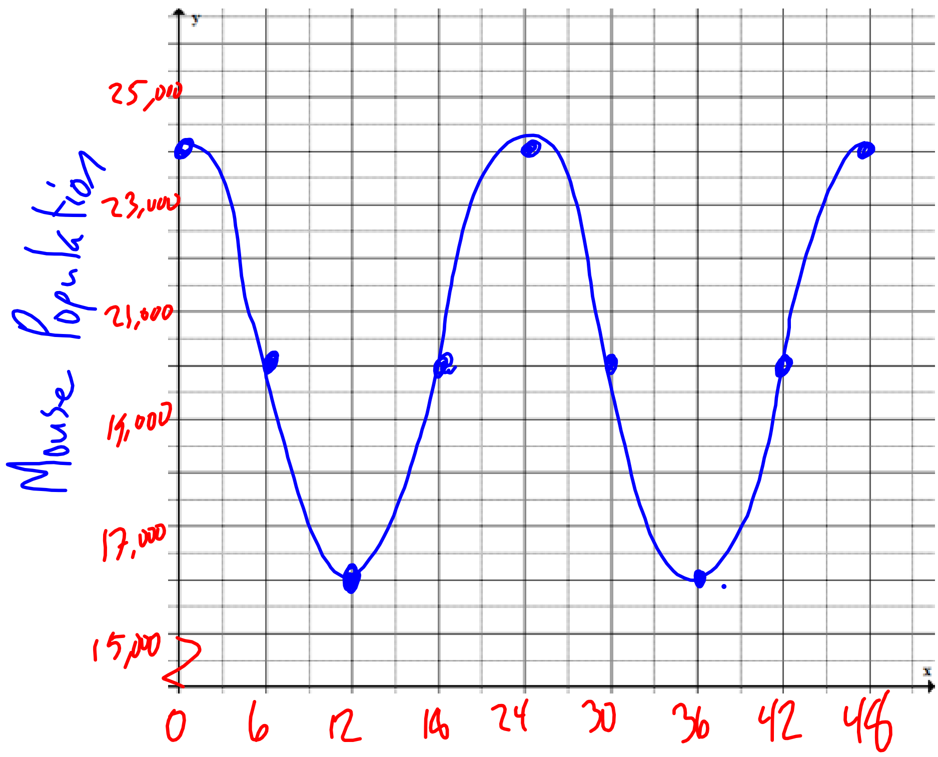
a) Sketch the graphs of these functions for the first 4 years. *go up by 6 months for each major grid line.
b) Compare the graphs, and discuss the relationships between the two populations.
c) How does the mice-to-owls ratio change over time?
d) When is there the most food per owl? When is it safest for the mice?

$$O(t) = 1000 + 100\sin\left(\frac{\pi t}{12}\right)$$



axis: $O = 1000$
amplitude: 100
max: 1100
min: 900
period: $\frac{2\pi}{\frac{\pi}{12}} = \cancel{2\pi} \times \frac{12}{\cancel{\pi}}$
$= 24$
initial value $= 1000$

$$m(t) = 20000 + 4000\left(\cos\frac{\pi t}{12}\right)$$



axis: $m = 20000$
amplitude $= 4000$
max $= 24000$
min $= 16000$
period $= 24$ mos.
initial value $= 24000$

Time	Mice	Owls	M-to-O Ratio
0	24 000	1000	24:1
6	20 000	1100	18:1
12	16 000	1000	16:1
18	20 000	900	22:1
24	24 000	1000	24:1

24000/1000

d) i) at time 0, 24, ...

ii) When M:O ratio is highest, same time as when owls have most food.

Pg. 360

1, 2, 6, 7, 10