Minds On

Look at the graph for example 1.

Discuss with your partner what is happening at various stages of the graph.

You do not have to do any calculations, but you should discuss relative speeds.

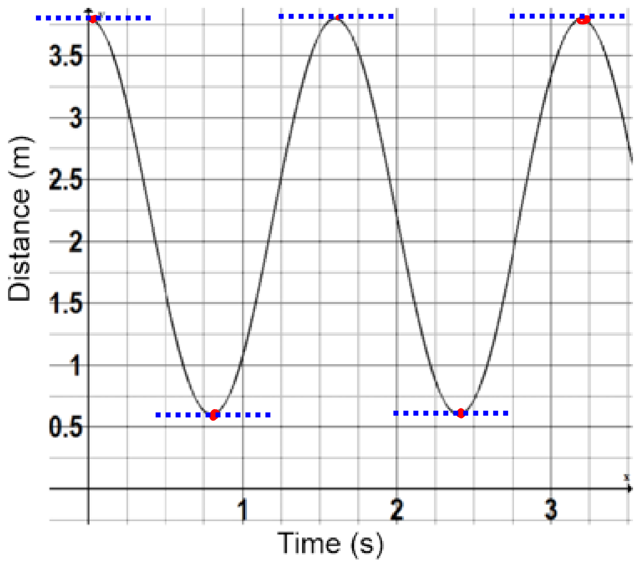
Action

Rates of Change in Trigonometric Functions

Example 1: Melissa used a motion detector to measure the horizontal distance between her and child on a swing. She stood in front of the child and recorded the distance, d(t), in metres over a period of time, t, in seconds. The data she collected are given in the following tables and are shown on the graph below. Use the data and the graph to discuss how the speed of the child changed as the child swung back and forth.

Time(s)	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1	1.1
Distance (m)	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.72	0.6	0.72	1.07	1.59

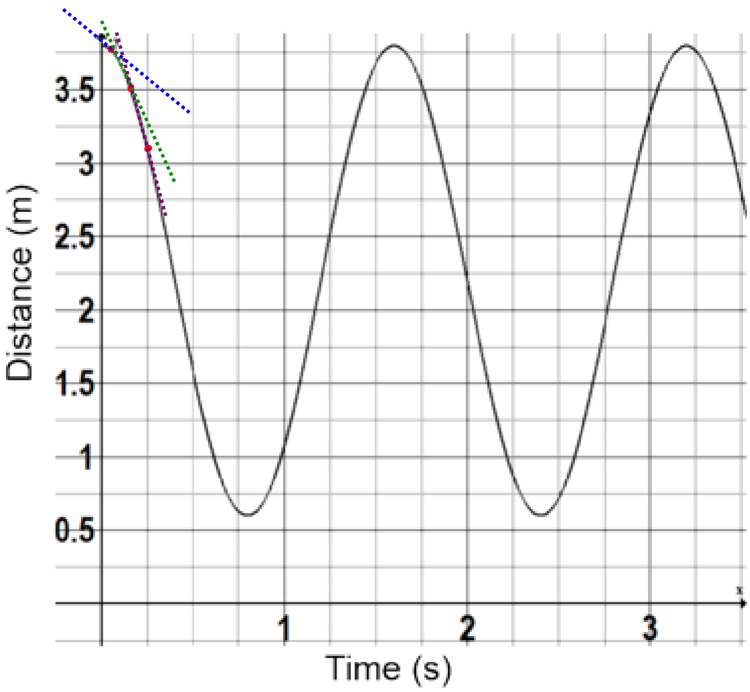
Time(s)	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3	2.4
Distance (m)	2.2	2.81	3.33	3.68	3.8	3.68	3.33	2.81	2.2	1.59	1.07	0.71	0.6



1 period = 1.6 s

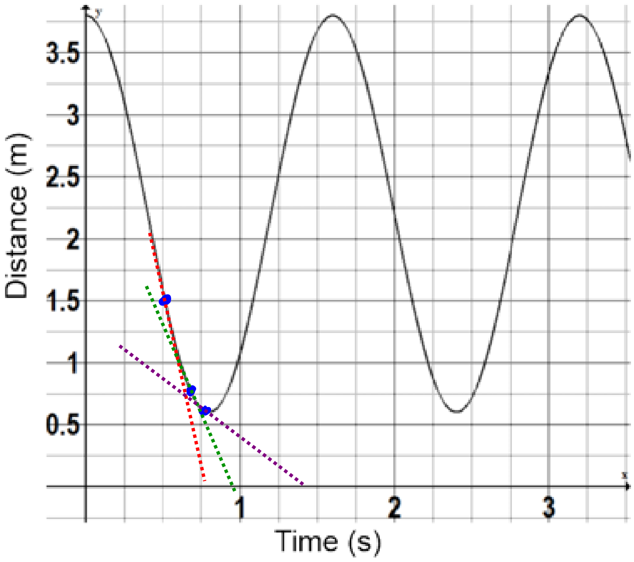
Farthest and Closest Points

the rate of change is 0



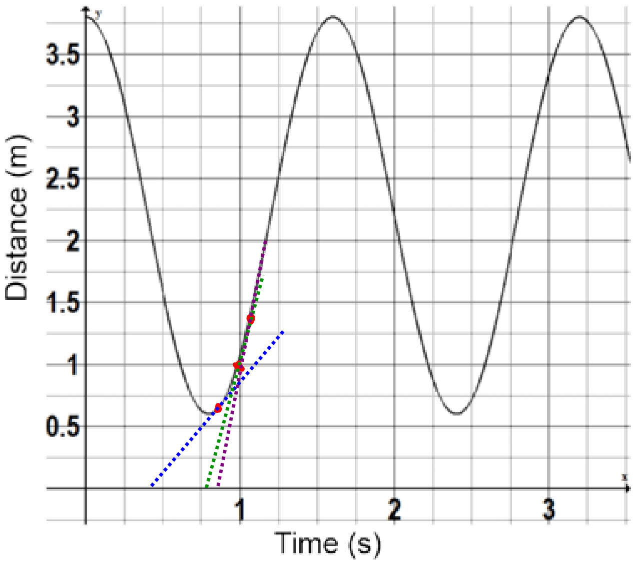
0 s to 0.4 s

As time increases, the tangent lines get steeper. The rate of change is negative and speed increasing.



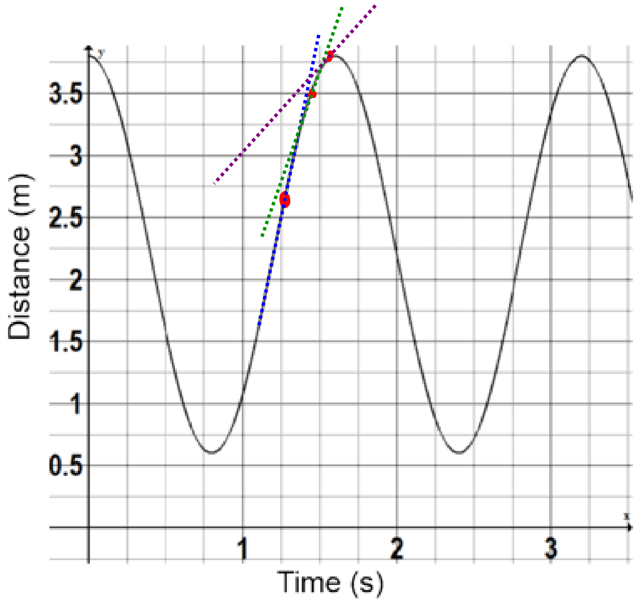
0.4 s to 0.8 s

As time increases, the tangent lines become less steep. Rate of change/ speed is negative and decreasing in magnitude



0.8 s to 1.2 s

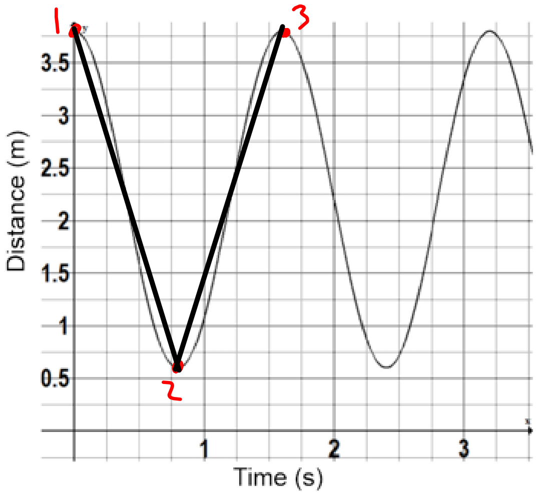
As time increases, the tangent lines are getting steeper. Rate of change is therefore increasing and positive.



1.2 s to 1.6 s

As time increases, ~~the~~ tangent lines become less steep. Rate of change is decreasing, the girl is getting farther away, but slowing down.

Example 2: Calculate the child's average speed over the intervals of time as the child swung toward and away from the motion detector on the first swing.



Moving Towards

Find the slope of the secant line between points 1 and 2.

$$\frac{\Delta y}{\Delta x} = \frac{0.6-3.8}{0.8-0}$$

$$= -4$$

Average speed is -4 m/s

or 4 m/s towards mom.

Moving Away

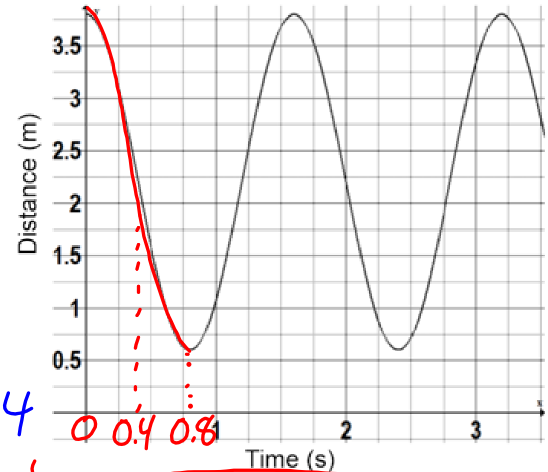
Find the slope of the secant line between points 2 and 3.

$$\frac{\Delta y}{\Delta x} = \frac{3.8-0.6}{1.6-0.8}$$

$$= 4$$

Average speed is 4 m/s away from mom.

Example 3: To model the motion of the child on the swing, Melissa determined that she could use the equation $d(t) = 1.6\cos\left(\frac{\pi}{0.8}t\right) + 2.2$, where d(t) is the distance from the child to the motion detector, in metres, and t is the time, in seconds. Use this equation to estimate when the child was moving the fastest and what speed the child was moving at this time.



We think speed is fastest at 0.4s.

Calculate instantaneous ROC when $t = 0.4$

$$ROC = \frac{f(0.4+0.001) - f(0.4)}{0.001}$$

$$= \frac{f(0.401) - f(0.4)}{0.001}$$

$$= \frac{2.1937 - 2.2}{0.001}$$

$$= \frac{-0.0063}{0.001}$$

$$= -6.3 \text{ m/s}$$

graphing calculator

$0.1 \rightarrow -2.4$
$0.2 \rightarrow -4.4$
$0.3 \rightarrow -5.6$
$\boxed{0.4 \rightarrow -6.3}$
$0.5 \rightarrow -5.6$
$0.6 \rightarrow -4.4$
$0.7 \rightarrow -2.4$

The rate of change of a sinusoidal function is always *highest halfway between the maximum and minimum values.

*highest absolute value

Consolidation

For the function below, determine a point when the instantaneous rate of change is

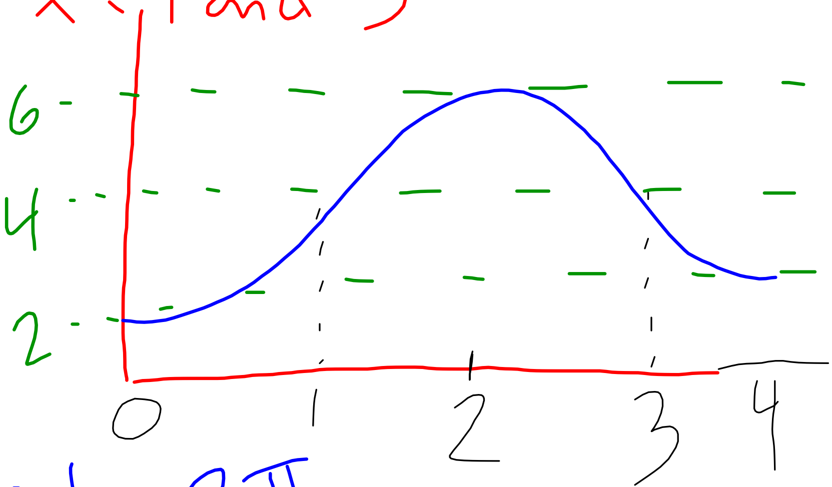
$$f(x) = -2\cos\left(\frac{\pi x}{2}\right) + 4$$

a) positive any point between x=0, x=2

b) negative any point between x=2, x=4

c) zero x = 0, 2, 4

d) greatest x = 1 and 3



$$\text{Period} = \frac{2\pi}{\frac{\pi}{2}}$$

$$= 2\cancel{\pi} \times \frac{2}{\cancel{\pi}}$$

$$= 4$$

Pg. 369

1, 2, 6, 8, 12