Learning Goal: I will identify equivalent trigonometric relationships.

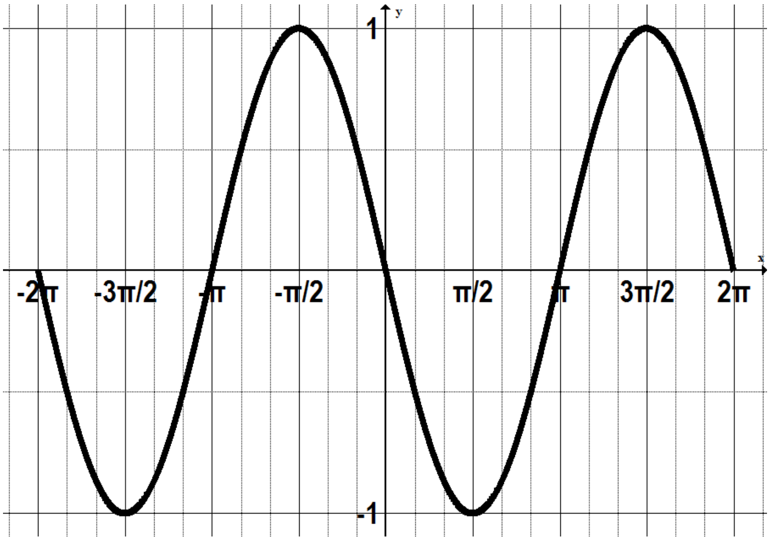
Minds On: What's the Equation?

Action: Equivalent Trigonometric Functions

Consolidation: Are we Equivalent?

Minds On

What's the Equation?



Write 4 possible equations for this graph.

$\boxed{-\sin\theta}$

$\sin(\theta - \pi)$

$\sin(\theta + \pi)$

$\cos(\theta + \frac{\pi}{2})$

$\sin(-\theta)$

$-\sin(\theta + 2\pi)$

$-\sin(\theta + 6\pi)$

Action

Even and Odd Functions

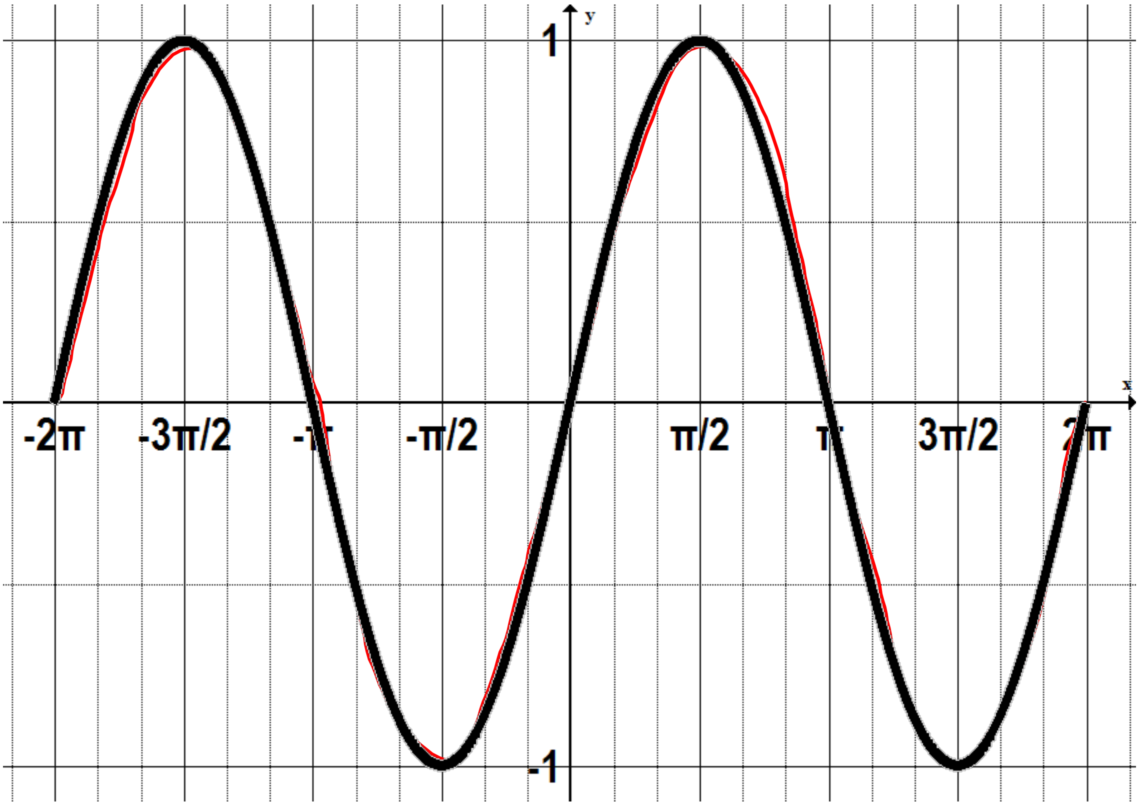
An **even function** is a function whose graph is symmetrical in the y-axis.

An **odd function** is a function whose graph has rotational symmetry about the origin.

Action

Even and Odd Functions

Is f(θ) = sin θ an even or odd function?



$\sin\theta = -\sin(-\theta)$

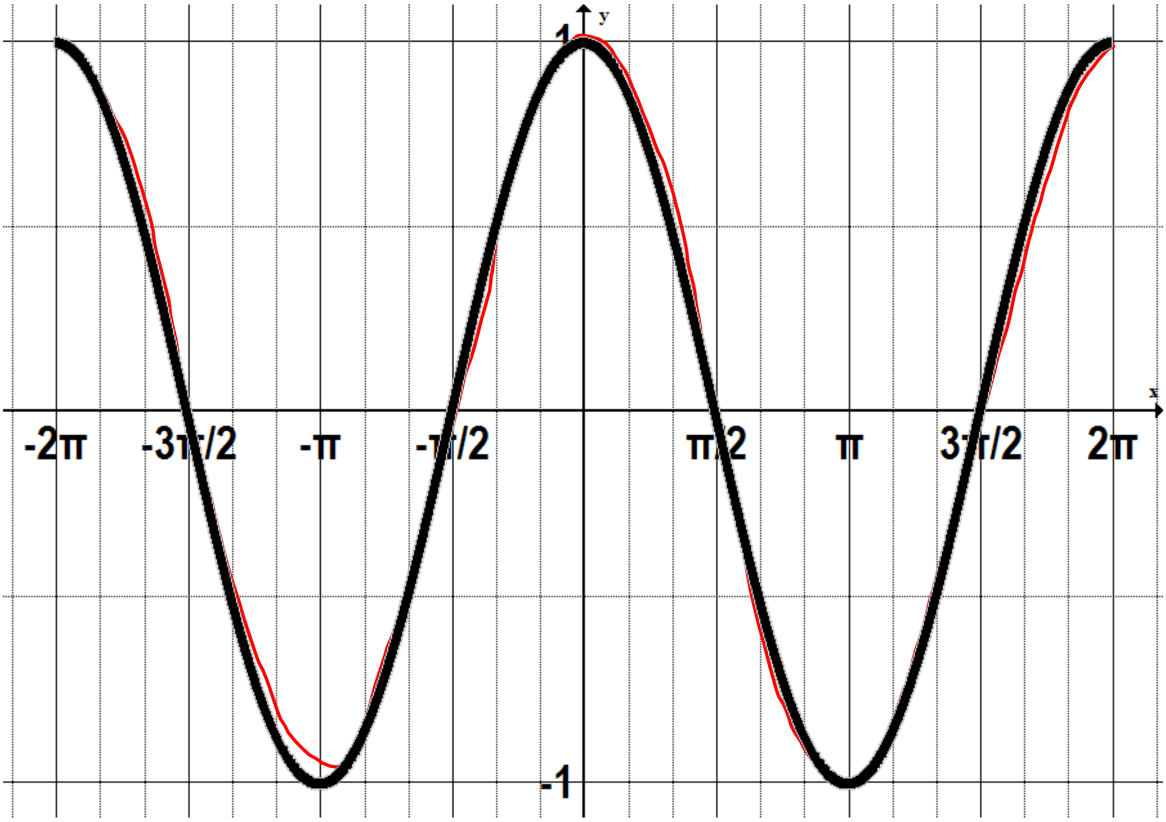
a=-1 (reflected in x axis)

k=-1 (reflected in y axis)

Action

Even and Odd Functions

Is f(θ) = cos θ an even or odd function?

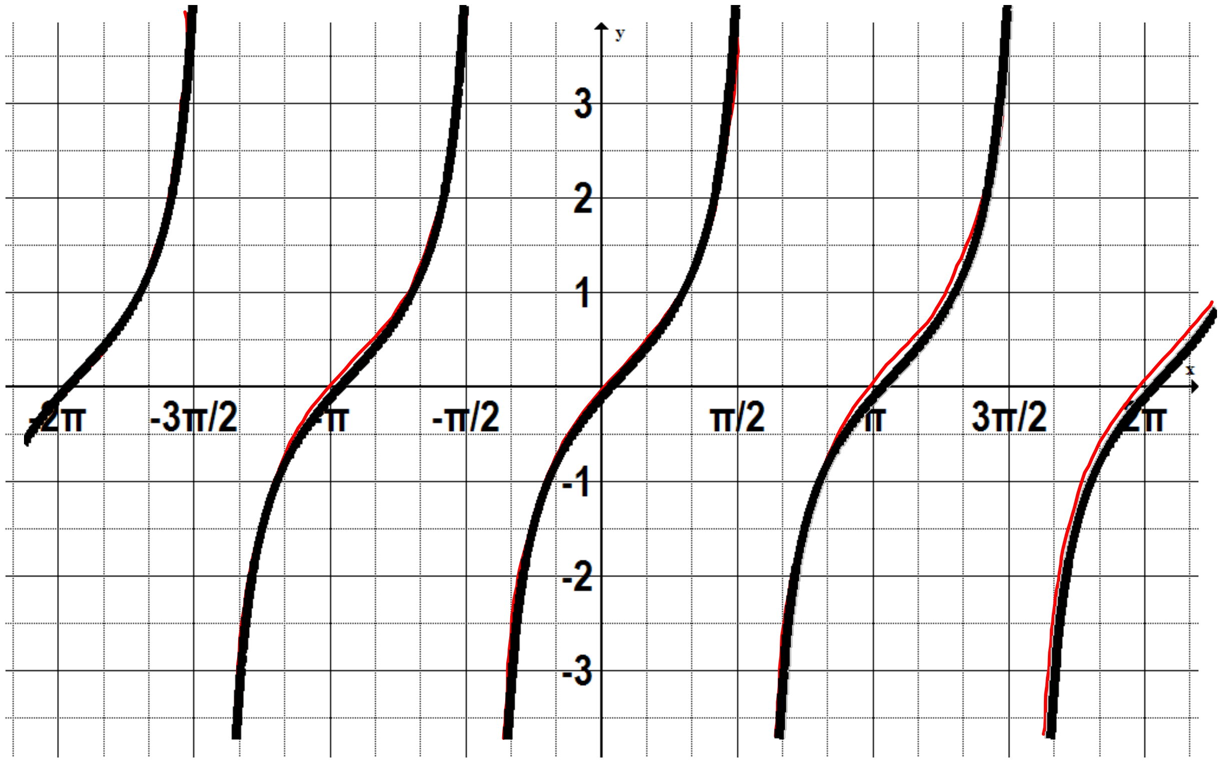


$\cos\theta = \cos(-\theta)$

Action

Even and Odd Functions

Is f(θ) = tan θ an even or odd function?



$\tan\theta = -\tan(-\theta)$

In general

Even Functions

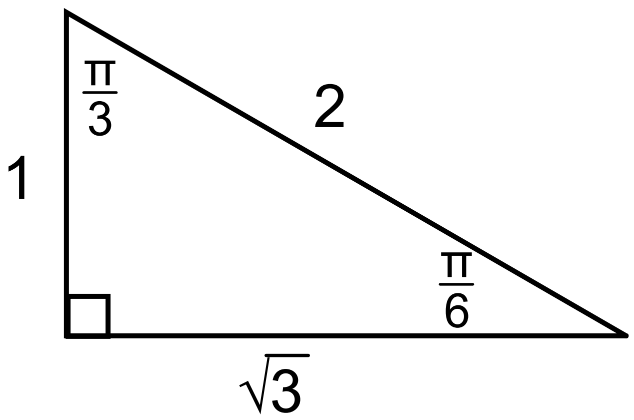
$f(x) = f(-x)$

Odd Functions

$f(x) = -f(-x)$

Action

Cofunction Identities



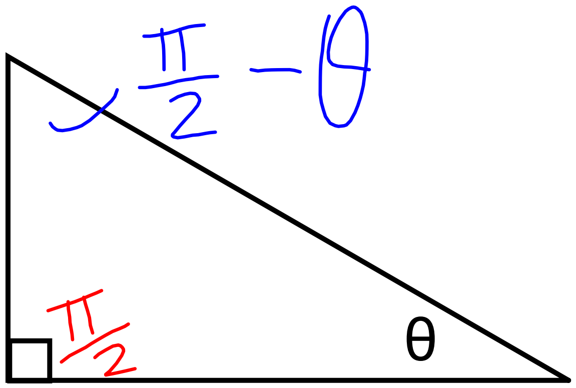
$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	$\csc\left(\frac{\pi}{3}\right) = \frac{2}{\sqrt{3}}$
$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	$\sec\left(\frac{\pi}{3}\right) = \frac{2}{1}$
$\tan\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{1}$	$\cot\left(\frac{\pi}{3}\right) = \frac{1}{\sqrt{3}}$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	$\csc\left(\frac{\pi}{6}\right) = \frac{2}{1}$
$\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	$\sec\left(\frac{\pi}{6}\right) = \frac{2}{\sqrt{3}}$
$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}}$	$\cot\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{1}$

What expressions are equivalent?

Action

Cofunction Identities



What is the measure of the missing angle?

Then:

sin θ = $\cos\left(\frac{\pi}{2} - \theta\right)$

cos θ = $\sin\left(\frac{\pi}{2} - \theta\right)$

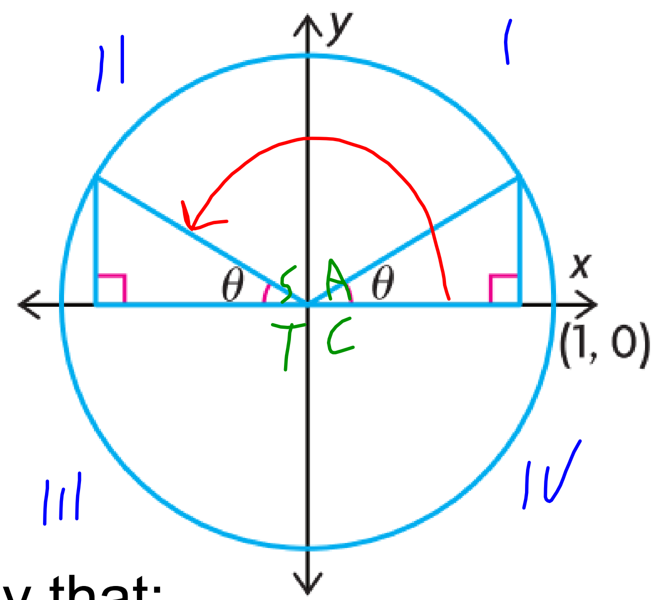
tan θ = $\cot\left(\pi/2 - \theta\right)$

Action

Equivalent Expressions from the Plane

With a related acute angle, θ, in quadrant I we can find more equivalent expressions.

Here, the angle of interest is (π - θ).



We can say that:

$\sin(\pi - \theta) = \sin(\theta)$

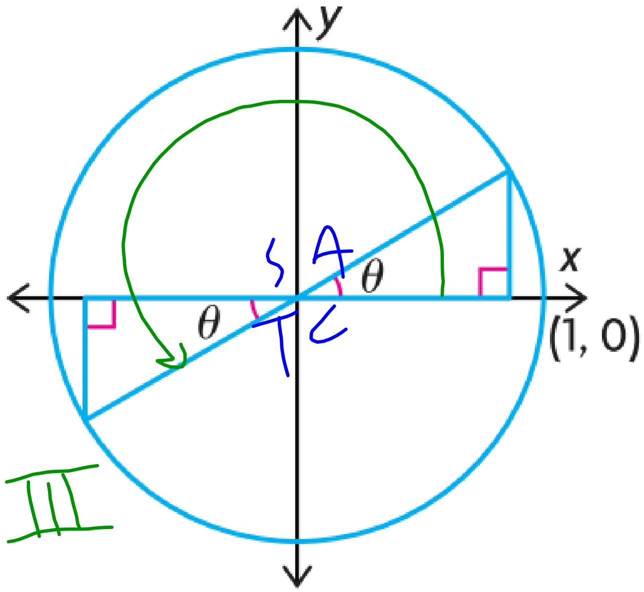
$\cos(\pi - \theta) = -\cos(\theta)$

$\tan(\pi - \theta) = -\tan(\theta)$

Action

Equivalent Expressions from the Plane

Here, the angle of interest is $\pi + \theta$.



We can say that:

$$\sin(\pi+\theta) = -\sin(\theta)$$

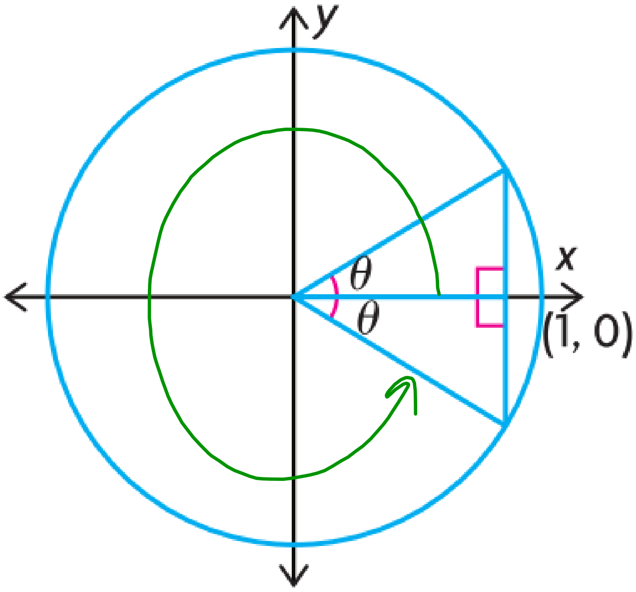
$$\cos(\pi+\theta) = -\cos(\theta)$$

$$\tan(\pi+\theta) = \tan(\theta)$$

Action

Equivalent Expressions from the Plane

Here, the angle of interest is $2\pi - \theta$.



We can say that:

$$\sin(2\pi - \theta) = -\sin(\theta)$$
$$\cos(2\pi - \theta) = \cos(\theta)$$
$$\tan(2\pi - \theta) = -\tan(\theta)$$

Action

Producing Equivalent Expressions

We can produce equivalent trigonometric expressions by:

1. Horizontal Translations

   by multiples of the period $(\pm 2\pi, 4\pi, \ldots)$

   by shifting sin and cos by pi/2

2. Reflecting **even functions** across y-axis

   cos (x) = cos (-x)

3. Reflecting **odd functions** across x-axis and y-axis

   sin x = - sin (-x)

   - sin x = sin (-x)

   tan x = - tan (-x)

   -tan x = tan (-x)

Action

Producing Equivalent Expressions

We can produce equivalent trigonometric expressions by:

4. Using the cofunction identities

$\sin \theta = \cos (\pi/2 - \theta)$

$\cos \theta = \sin (\pi/2 - \theta)$

$\tan \theta = \cot (\pi/2 - \theta)$

Action

Producing Equivalent Expressions

We can produce equivalent trigonometric expressions by:

5. Using principle and related acute angles

In Quadrant II

$\sin(\pi - \theta) = \sin\theta$

$\cos(\pi - \theta) = -\cos\theta$

$\tan(\pi - \theta) = -\tan\theta$

In Quadrant III

$\sin(\pi + \theta) = -\sin\theta$

$\cos(\pi + \theta) = -\cos\theta$

$\tan(\pi + \theta) = \tan\theta$

In Quadrant IV

$\sin(2\pi - \theta) = -\sin\theta$

$\cos(2\pi - \theta) = \cos\theta$

$\tan(2\pi - \theta) = -\tan\theta$

Consolidation

Are They Equivalent?

Does $\cos(\theta + \pi) = \cos\theta$?

***Use the unit circle.**

WILL DO WEDNESDAY

Consolidation

Are They Equivalent?

Does $\cos\left(\frac{\pi}{2} - \theta\right) = -\cos\left(\theta - \frac{\pi}{2}\right)$?

***Use the unit circle.**

WILL DO WEDNESDAY

Consolidation

Are They Equivalent?

Does $\cos\left(\frac{\pi}{2} - \theta\right) = \cos\frac{\pi}{2} - \cos\theta$?

***Use the unit circle.**

WILL DO WEDNESDAY

Consolidation

Practice Questions

Pg. 392

1 - 3, 5, 7