Learning Goal: I will be able to use compound angle formulas to determine exact values of trig ratios.

Minds On: Checking compound angle formulas

Action: Note and practice

Consolidation: Exit Question

Online Solution Manual

If you search

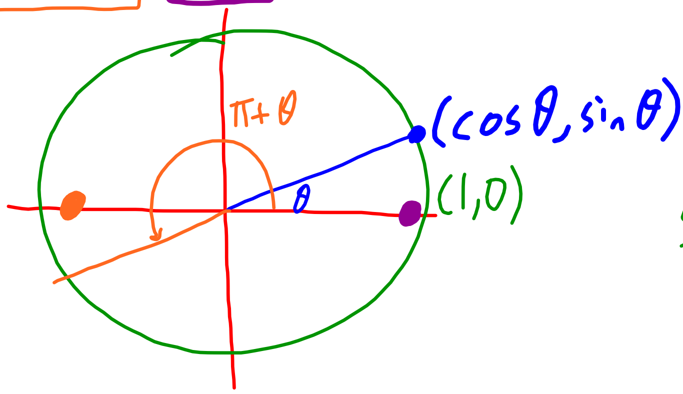
"advanced functions nelson solutions"

the second result is a pdf file of all the solutions to the problems in our textbook.

Equivalent or Not?

Using the unit circle, determine whether each of the following statements are true.

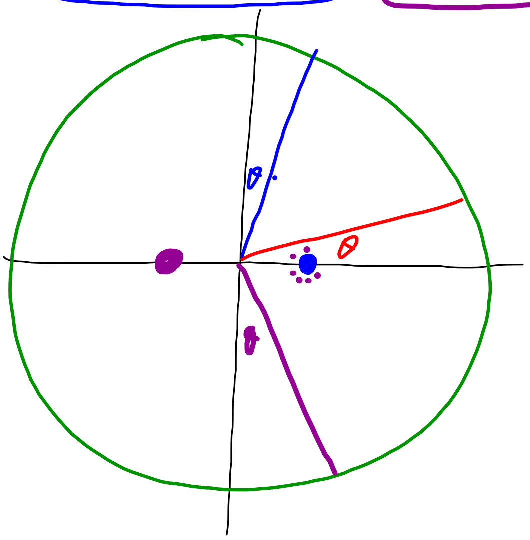
1. $\boxed{\cos(\theta + \pi)} = \boxed{\cos\theta}$



The points are not the same,

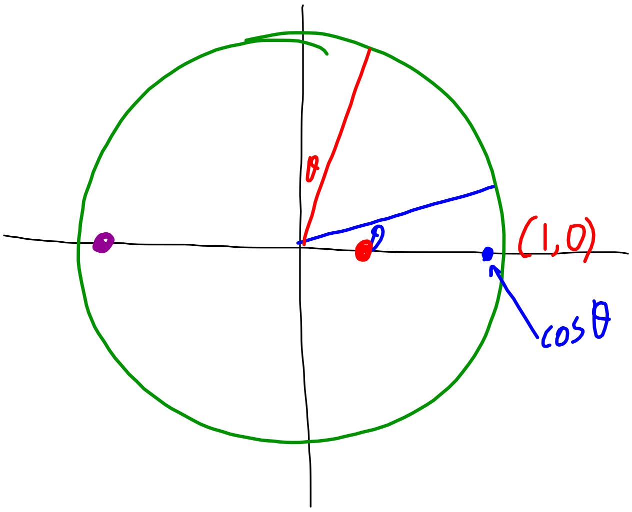
so $\cos(\theta + \pi) \neq \cos\theta$

2. $\boxed{\cos\left(\frac{\pi}{2} - \theta\right)} = \boxed{-\cos\left(\theta - \frac{\pi}{2}\right)}$



$\cos\left(\frac{\pi}{2} - \theta\right) \neq -\cos\left(\theta - \frac{\pi}{2}\right)$

3. $\boxed{\cos\left(\frac{\pi}{2}-\theta\right)} = \boxed{\cos\frac{\pi}{2}-\cos\theta}$



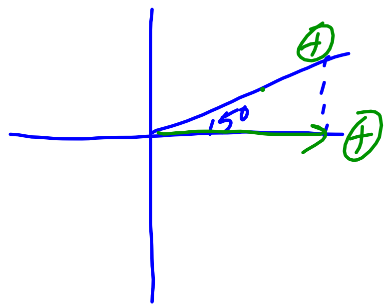
$\cos\left(\frac{\pi}{2}-\theta\right) \neq \cos\frac{\pi}{2}-\cos\theta$

Minds On

Exploring cos (a - b)

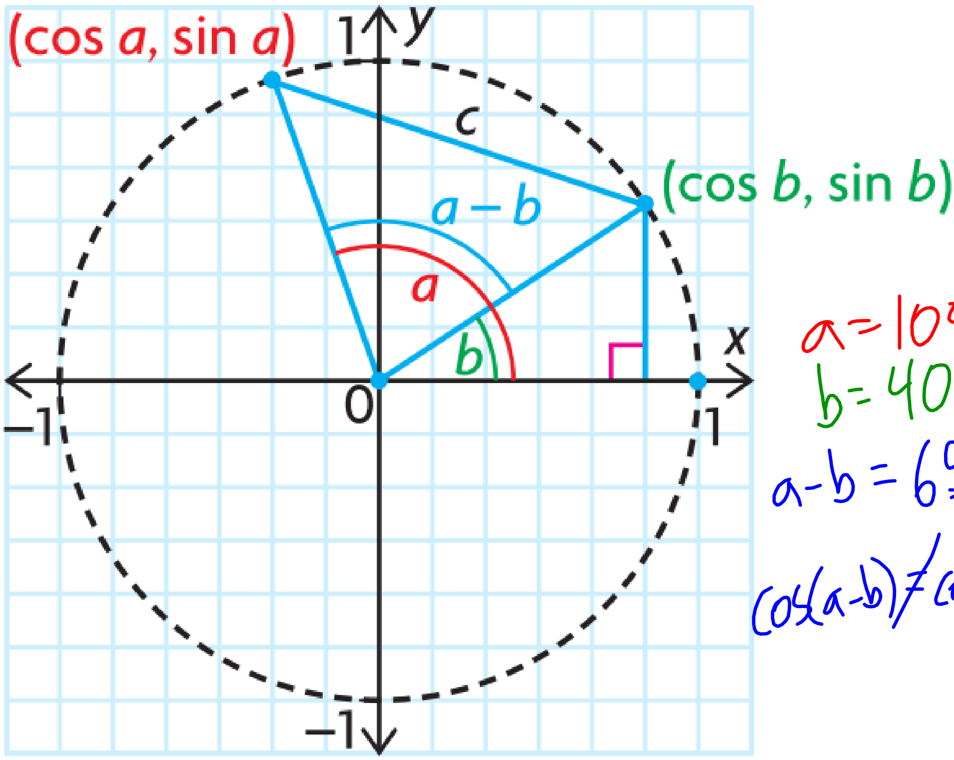
Given that $\cos 45^o = \frac{1}{\sqrt{2}}$ and $\cos 30^o = \frac{\sqrt{3}}{2}$, try and determine the exact value of $\cos 15^o$. Check your answer using a calculator.

$$\cos 15^\circ = \cos 45^\circ - \cos 30^\circ$$
$$= \frac{1}{\sqrt{2}} - \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{2} - \sqrt{3}}{2}$$
$$= -0.1589$$



Minds On

Determining cos (a - b)



$a = 105°$

$b = 40°$

$a - b = 65°$

$\cos(a-b) \neq \cos a - \cos b$

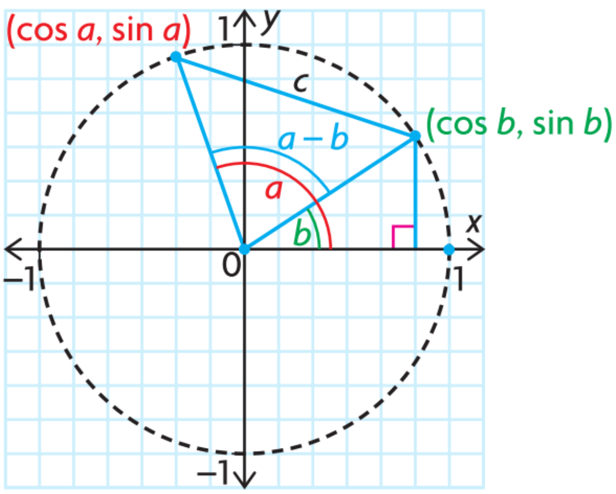
Things we know:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c = \sqrt{(\sin a - \sin b)^2 + (\cos a - \cos b)^2}$$

Minds On

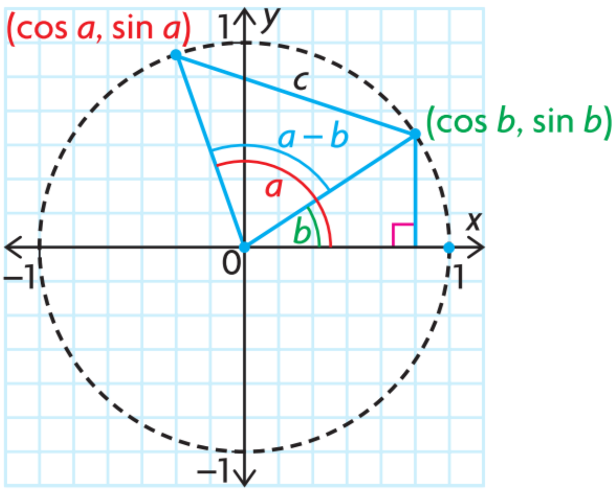
Determining cos (a - b)



$$c^2 = a^2 + b^2 - 2ab\cos C$$

Minds On

Determining cos (a - b)



$$c = \sqrt{(\sin a - \sin b)^2 + (\cos a - \cos b)^2}$$

Minds On

Determining cos (a + b)

Minds On

Compound Angle Formulas

Here are some new formulas called the Compound Angle Formulas:

Addition Formulas

sin (a + b) = sin a cos b + cos a sin b

cos (a + b) = cos a cos b – sin a sin b

tan (a + b) = $\frac{\tan a + \tan b}{1 - \tan a \tan b}$

Subtraction Formulas

sin (a – b) = sin a cos b – cos a sin b

cos (a – b) = cos a cos b + sin a sin b

tan (a – b) = = $\frac{\tan a - \tan b}{1 + \tan a \tan b}$

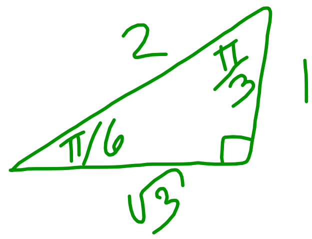
Minds On

<u>Minds On:</u> Show that each of the Compound Angle Formulas work for a = π/3 and b = π/6

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

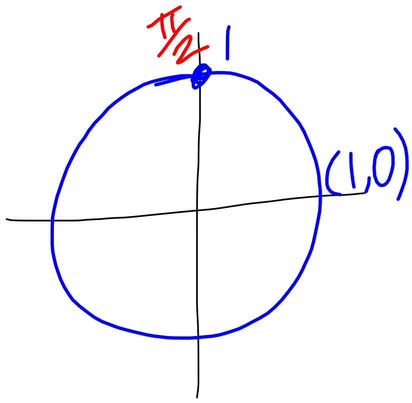
$$\sin\left(\frac{\pi}{3}+\frac{\pi}{6}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{6} + \cos\frac{\pi}{3}\sin\frac{\pi}{6}$$

$$\sin\left(\frac{\pi}{2}\right) = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{1}{2}$$

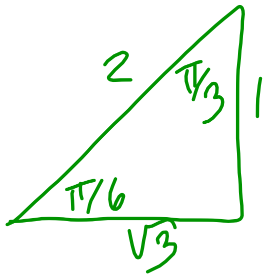


$$\sin\left(\frac{\pi}{2}\right) = \frac{3}{4} + \frac{1}{4}$$

$$\sin\left(\frac{\pi}{2}\right) = 1 \checkmark$$

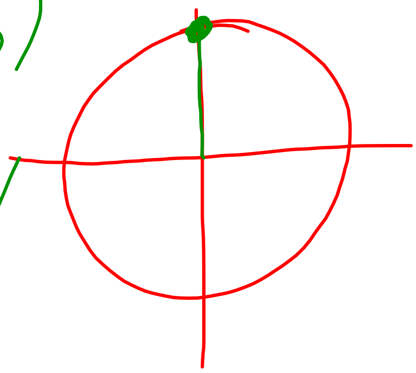


$$\tan\left(\frac{\pi}{3}+\frac{\pi}{6}\right) = \frac{\tan\frac{\pi}{3}+\tan\frac{\pi}{6}}{1-\tan\frac{\pi}{3}\tan\frac{\pi}{6}}$$



$$\tan\left(\frac{\pi}{2}\right) = \frac{\sqrt{3}+\frac{1}{\sqrt{3}}}{1-\left(\frac{\sqrt{3}}{1}\right)\left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3}+\frac{1}{\sqrt{3}}}{0}$$

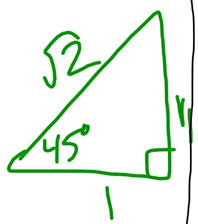


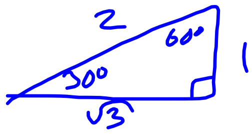
Action

Example 1: Determine the exact value of:

a) cos (15°)

$$= \cos(45° - 30°)$$





$$= \cos 45° \cos 30° + \sin 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

b) $\tan\left(-\frac{5\pi}{12}\right)$

Convert to degrees

$$\frac{-5\pi}{12} \times \frac{180}{\pi} = -75°$$

$$\tan(-75) = (-30° - 45°)$$

$\tan(a+b)$ where $a = -30$, $b = -45$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

tan(-30) + tan(-45) are negative

$$= \frac{\tan(-30) + \tan(-45)}{1 - (\tan(-30))(\tan(-45))}$$

$$= \frac{\left(-\frac{1}{\sqrt{3}}\right) + (-1)}{1 - \left(-\frac{1}{\sqrt{3}}\right)(-1)}$$

Continued on next page

$$= \frac{\left(-\frac{1}{\sqrt{3}}\right) + (-1)}{1 - \left(-\frac{1}{\sqrt{3}}\right)(-1)}$$

$$= \frac{\left(-\frac{1}{\sqrt{3}}\right) + \left(-1 \cdot \frac{\sqrt{3}}{\sqrt{3}}\right)}{1 - \left(\frac{1}{\sqrt{3}}\right)}$$

$$= \frac{-\frac{1}{\sqrt{3}} + \frac{-\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{-1-\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$$

$$= \frac{-1-\sqrt{3}}{\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{\sqrt{3}-1}$$

$$= \frac{-\sqrt{3}-1}{\sqrt{3}-1}$$

Something from yesterday.

We were asked whether this statement was true.

$$\cos(a - b) \neq \cos a - \cos b$$

McKenna asked if we could just rewrite the right

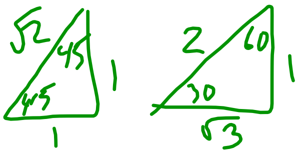
side as $\cos\left(\frac{\pi}{2} - \theta\right)$ sort of like factoring.

I said no, we now have the tools to explain why the answer is no.

How do we know the statement is false without using quadrants or the unit circle?

Minds On

Determine the exact value of each trig ratio:

a) cos 105° 

b) $\tan \frac{\pi}{12}$

Special angles: 30°, 45°, 60°, 90°

$\cos(45° + 60°)$

$= \cos a \cos b - \sin a \sin b$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right)$

$\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{1}{2\sqrt{2}} - \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

$= \frac{1 - \sqrt{3}}{2\sqrt{2}}$

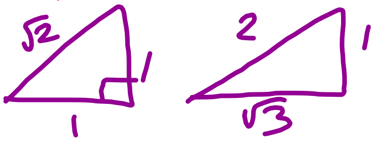
b) $\tan \frac{\pi}{12}$

$\frac{\pi}{12} = 15°$

$15° = 45° - 30°$

or

$15° = 60° - 45°$



$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \dfrac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$= \dfrac{\sqrt{3} - 1}{1 + (\sqrt{3})(1)}$

$= \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$\tan\left(\frac{\pi}{12}\right) = \tan\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$

$= \dfrac{\tan\frac{\pi}{4} - \tan\frac{\pi}{6}}{1 + \tan\frac{\pi}{4}\tan\frac{\pi}{6}}$

$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + (1)\left(\frac{1}{\sqrt{3}}\right)}$

$= \dfrac{\frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{\sqrt{3} - 1}{\sqrt{3}}}{\frac{\sqrt{3} + 1}{\sqrt{3}}}$ *flip & multiply and √3 cancels

$= \dfrac{\sqrt{3} - 1}{\sqrt{3} + 1}$

Action

Example 2: Simplify each expression.

a) $\cos\frac{7\pi}{12}\cos\frac{5\pi}{12}+\sin\frac{7\pi}{12}\sin\frac{5\pi}{12}$

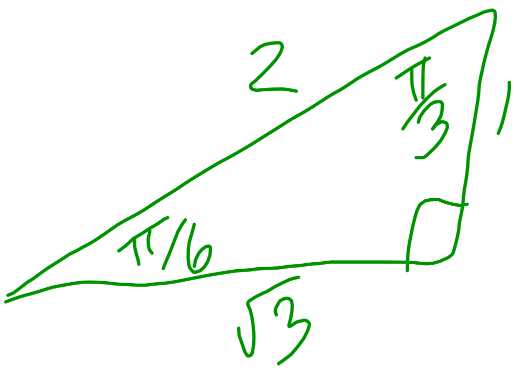
$\cos a\cos b+\sin a\sin b=\cos(a-b)$

$=\cos\left(\frac{7\pi}{12}-\frac{5\pi}{12}\right)$

$=\cos\left(\frac{2\pi}{12}\right)$

$=\cos\left(\frac{\pi}{6}\right)$

$=\frac{\sqrt{3}}{2}$



Action

b) sin 2x cos x – cos 2x sin x

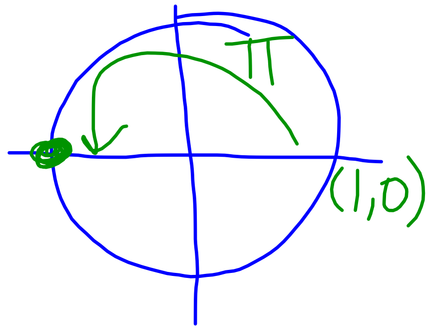
$\sin(a-b)$

$= \sin(2x - x)$

$= \sin(x)$

Action

Example 3: Use compound angle formulas to show that sin $(x-\pi)$, sin $(x+\pi)$, and cos $(x+\frac{\pi}{2})$ are equivalent trig expressions.



$$\sin(x-\pi)$$

$$= \sin x \cos\pi - \cos x \sin\pi$$

$$= \sin x(-1) - \cancel{\cos x(0)}$$

$$= -\sin x$$

$\sin(x+\pi)$

$= \sin x \cos\pi + \cos x \sin\pi$

$= -\sin x$

$\cos(x+\frac{\pi}{2})$

$= \cos x \cos\frac{\pi}{2} - \sin x \sin\frac{\pi}{2}$

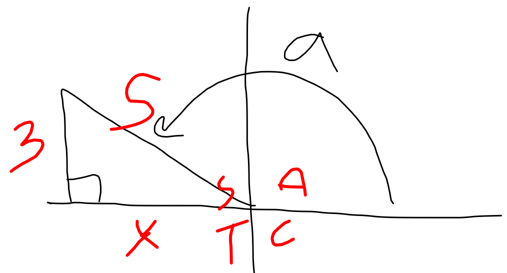
$= \cos x(0) - \sin x(1)$

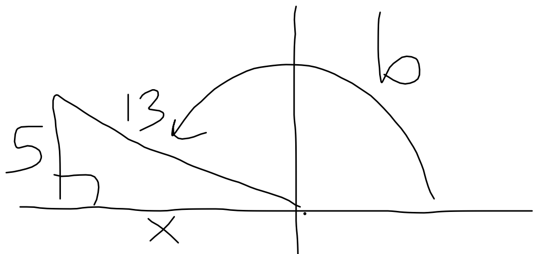
$= -\sin x$

Action

Example 4: Evaluate sin (a + b), where a and b are obtuse angles; sin a = $\frac{3}{5}$ and sin b = $\frac{5}{13}$.

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$





$$x^2 = 5^2 - 3^2$$
$$\sqrt{x^2} = \sqrt{16}$$
$$x = 4$$

$$x^2 = 13^2 - 5^2$$
$$x = 12$$

$$\cos a = \frac{-4}{5}$$

$$\cos b = \frac{-12}{13}$$

$$\sin(a+b) = \left(\frac{3}{5}\right)\left(\frac{-12}{13}\right) + \left(\frac{-4}{5}\right)\left(\frac{5}{13}\right)$$
$$= \frac{-36}{65} + \frac{-20}{65}$$
$$= \frac{-56}{65}$$

Determine the exact value of tan(75°)

$$\tan 75^\circ = \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

FOIL

$$= \frac{3+\sqrt{3}+\sqrt{3}+1}{3+\cancel{\sqrt{3}}-\cancel{\sqrt{3}}-1}$$

$$= \frac{4+2\sqrt{3}}{2}$$

$$= 2+\sqrt{3}$$

Consolidation

Practice

Wednesday

Pg. 400

1, 3, 4, 5, 8

Thursday

Pg. 400

2, 6, 10 - 13