Learning Goal: I will develop and use double angle formulas

Minds On: Same or different?

Action: Double Angle Formulas - Investigation and Practice

Consolidation: Exit Question

Minds On

Determine the exact value of the expression below

$$\tan(30^o - 45^o)$$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\tan 45° = 1$

$$\tan(30°-45°) = \frac{\tan 30° - \tan 45°}{1 + \tan 30° \tan 45°}$$

$$= \frac{\frac{1}{\sqrt{3}} - 1 \cdot \frac{\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} \cdot 1 + \left(\frac{1}{\sqrt{3}}\right)(1)}$$

$$= \frac{\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{3}}}{\frac{\sqrt{3}}{\sqrt{3}} + \frac{1}{\sqrt{3}}}$$

$$= \frac{\frac{1-\sqrt{3}}{\cancel{\sqrt{3}}}}{\frac{1+\sqrt{3}}{\cancel{\sqrt{3}}}}$$

$$= \frac{(1-\sqrt{3})}{(1+\sqrt{3})} \times \frac{(1-\sqrt{3})}{(1-\sqrt{3})}$$

$$= \frac{1-\sqrt{3}-\sqrt{3}+3}{1-\cancel{\sqrt{3}}+\cancel{\sqrt{3}}-3}$$

$$= \frac{4-2\sqrt{3}}{-2}$$

$$= \frac{2-\sqrt{3}}{-1}$$

$$= \sqrt{3}-2$$

Minds On

Same or Different?

Describe $f(x) = \boxed{\sin(2x)}$ and $f(x) = \boxed{2\sin(x)}$

Are they the same or different?

Action

Investigation:

1. Given $\sin 2\theta = \sin(\theta + \theta)$, use the appropriate compound angle formula to expand the right side of the equation. Simplify both sides to develop a formula for $\sin 2\theta$.
2. Verify your double angle formula for sine by graphing each side of the formula as a function.

$$\sin(\theta+\theta) = \sin\theta\cos\theta + \cos\theta\sin\theta$$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

3. Repeat parts 1 and 2 to develop a double angle formula for $\tan 2\theta$.

$$\tan(2\theta) = \tan(\theta + \theta)$$

$$\tan(\theta + \theta) = \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$

$$= \frac{2\tan\theta}{1 - (\tan\theta)^2}$$

$$= \frac{2\tan\theta}{1 - \tan^2\theta}$$

4. Repeat parts 1 and 2 to develop a double angle formula for $\cos 2\theta$.

$$\cos 2\theta = \cos(\theta + \theta)$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

5. Use the Pythagorean Identity to rearrange your formula for $\cos 2\theta$ to create different versions of the double angle formula. Be sure to verify each new formula.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - (1 - \cos^2\theta)$$

$$= \cos^2\theta - 1 + \cos^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos^2\theta = 1 - \sin^2\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= (1 - \sin^2\theta) - \sin^2\theta$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

Action

Example 1: Simplify each of the following expressions and then evaluate.

a) $2sin\frac{\pi}{8}cos\frac{\pi}{8}$

$= \sin 2\left(\frac{\pi}{8}\right)$

$= \sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{2}}{2}$

b) $\frac{2tan\frac{\pi}{6}}{1-tan^2\frac{\pi}{6}}$

$= \tan 2\left(\frac{\pi}{6}\right)$

$= \tan\frac{\pi}{3}$

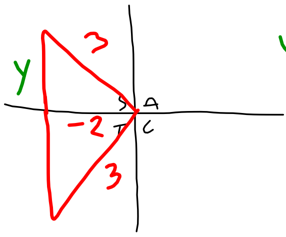
$= \sqrt{3}$

Action

<u>Example 2:</u> If $\cos\theta = -\frac{2}{3}$ and $0 \le \theta \le 2\pi$, determine the value of $\cos 2\theta$ and $\sin 2\theta$.

$$\cos(2\theta) = 2\cos^2\theta - 1$$
$$= 2\left(-\frac{2}{3}\right)^2 - 1$$
$$= 2\left(\frac{4}{9}\right) - 1$$
$$= \frac{8}{9} - 1 \cdot \frac{9}{9}$$
$$= \frac{-1}{9}$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$



$$y^2 = 3^2 - (-2)^2$$
$$y^2 = 5$$
$$y = \sqrt{5}$$

In QII $\quad \sin\theta = \frac{\sqrt{5}}{3}$

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{-2}{3}\right)$$
$$= \frac{-4\sqrt{5}}{9}$$

In QIII $\quad \sin\theta = \frac{-\sqrt{5}}{3}$

$$\sin 2\theta = 2\left(\frac{-\sqrt{5}}{3}\right)\left(\frac{-2}{3}\right)$$
$$= \frac{4\sqrt{5}}{9}$$

Action

<u>Example 3:</u> Given $\tan\theta = -\frac{3}{4}$ where $\frac{3\pi}{2} \leq \theta \leq 2\pi$, calculate the value of $\cos 2\theta$.

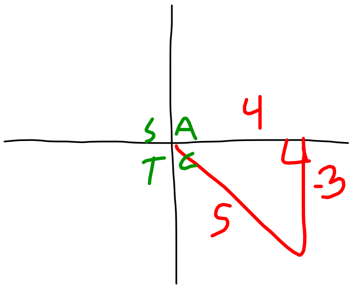
$$\boxed{\begin{aligned} \tan\theta &= -\frac{3}{4} \\ \sin\theta &= -\frac{3}{5} \\ \cos\theta &= \frac{4}{5} \end{aligned}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{4}{5}\right)^2 - \left(-\frac{3}{5}\right)^2$$

$$= \frac{16}{25} - \frac{9}{25}$$

$$= \frac{7}{29}$$



Action

Example 4: Develop a formula for $sin\frac{x}{2}$. $\theta = \frac{x}{2}$

$$\cos\left(2\left(\frac{x}{2}\right)\right) = 1 - 2\sin^2\frac{x}{2}$$

$$\cos x = \cancel{1} - 2\sin^2\frac{x}{2}$$

$$-1 \qquad \cancel{+1}$$

$$\frac{\cos x - 1}{-2} = \frac{-\cancel{2}\sin^2\frac{x}{2}}{-\cancel{2}}$$

$$\sqrt{\frac{1-\cos x}{2}} = \sqrt{\sin^2\left(\frac{x}{2}\right)}$$

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

Consolidation

Determine the value of $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$,

given $sin\theta = -\frac{12}{13}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$.

Tomorrow first thing

Consolidation

Pg. 407

2, 4, 6, 9, 11