Learning Goal: I will solve linear and quadratic trig equations.

Minds On: Solving equations without trig, how do periods affect our answers?

Action: Solving Linear Equations - Note

Consolidation: Exit Question

Minds On

Part 1: Solve Each Equation, Find All Values

$3(x + 1) + 5 = 2$

$3(x+1) = -3$

$x+1 = -1$

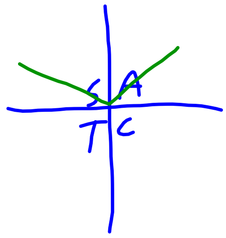
$x = -2$

$\cancel{3x^2 + 1 = 13}$

sin x = 0.4 (find in radians)

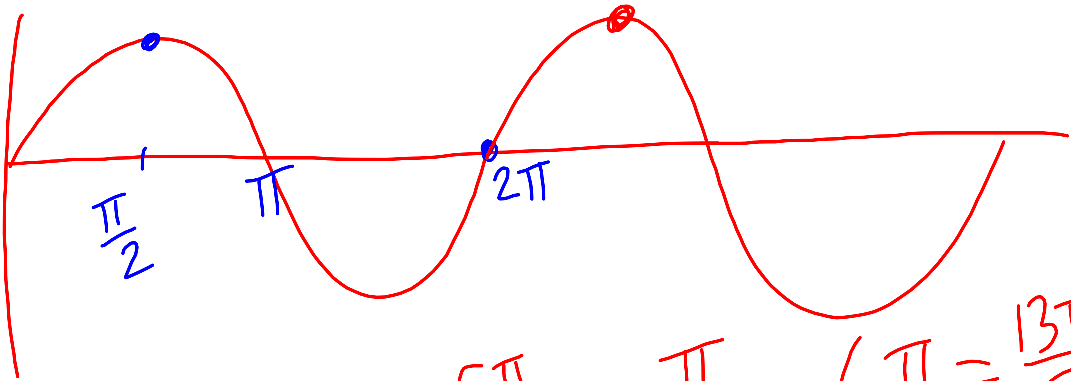
$x = 0.41$

$x = \pi - 0.41$



Part 2: Looking at trig periods

List 5 values where sin x = 1
How did you find them?



$\frac{\pi}{2} + 2\pi = \frac{5\pi}{2}$

$\frac{\pi}{2} + 6\pi = \frac{13\pi}{2}$

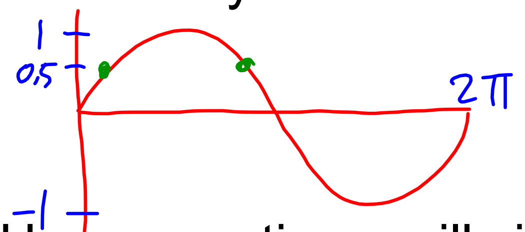
$\frac{\pi}{2} + 4\pi = \frac{9\pi}{2}$

$\frac{\pi}{2} + 8\pi = \frac{17\pi}{2}$

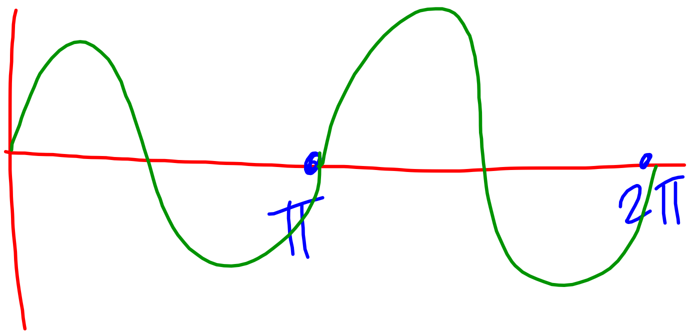
Minds On

Part 3: How many times?

How many times will sin(x) = 0.5 between 0 and 2π?



How many times will sin(2x) = 0.5 between 0 and 2π?



Action

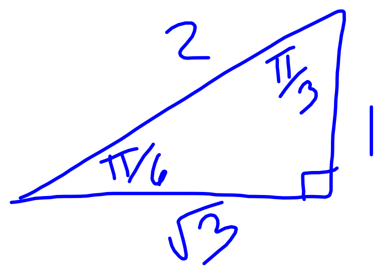
Solving Linear Trig Equations

Example 1: You are given the equation 2sinx + 1 = 0, 0 ≤ x ≤ 2π

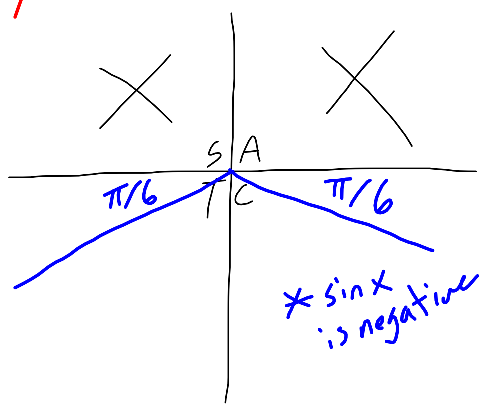
a) Determine all the solutions in the specified interval.

b) Verify the solutions using graphing technology

$$2\sin x + 1 = 0$$
$$-1 \quad -1$$
$$\frac{2\sin x}{2} = \frac{-1}{2}$$
$$\sin x = \frac{-1}{2}$$
$$x = \sin^{-1}\left(\frac{-1}{2}\right)$$



Related acute angle $= \frac{\pi}{6}$



$$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$$
$$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

Action

Example 2: Solve 3(tanx + 1) = 2 where 0° ≤ x ≤ 360°, to 1 decimal place

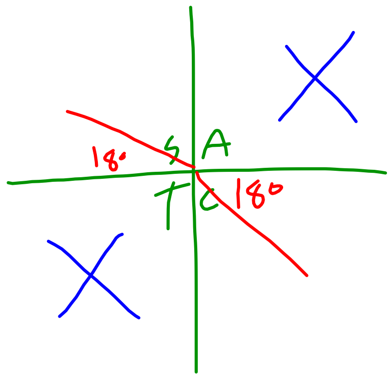
$$\frac{3(\tan x + 1)}{3} = \frac{2}{3}$$

$$\tan x + 1 = \frac{2}{3}$$

$$\tan x = \frac{2}{3} - 1$$

$$\tan x = \frac{-1}{3}$$

$$x = \tan^{-1}\left(\frac{-1}{3}\right)$$



Find related acute angle.

$$x = \tan^{-1}\left(\frac{1}{3}\right)$$

$$x = 18.4$$

Our angles are $180° - 18.4° = \boxed{161.6°}$
$360° - 18.4° = \boxed{341.6°}$

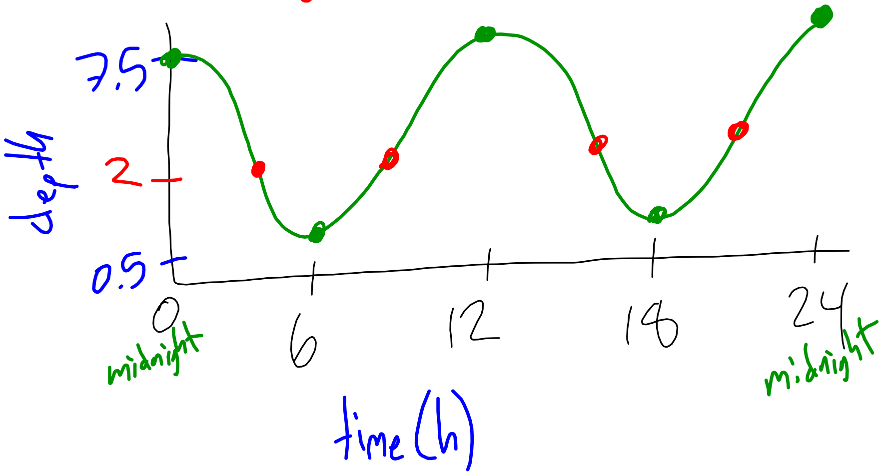
Action

Example 3: Today, the high tide in Matthews Cove, New Brunswick, is at midnight. The water level at high tide is 7.5 m. The depth, d metres, of the water in the cove at time t hours is modelled by the equation $d(t) = 4 + 3.5\cos(\frac{\pi}{6}t)$. Jenny is planning a day trip to the cove tomorrow, but the water need to be at least 2 m deep for her to manoeuvre her sailboat safely. How can Jenny determine the times when it will be safe for her to sail into Matthews cove?

axis: 4

max/min: 7.5/0.5 because amplitude is 3.5

Period: $\frac{2\pi}{\frac{\pi}{6}} = 12$ hours



$d(t) = 4 + 3.5\cos(\frac{\pi}{6}t)$

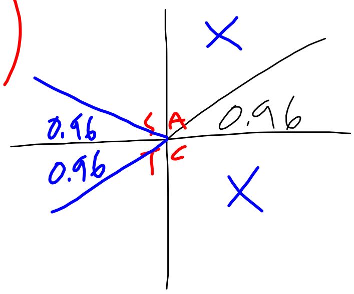
$4 + 3.5\cos(\frac{\pi}{6}t) = 2$

$3.5\cos(\frac{\pi}{6}t) = -2$

$\cos(\frac{\pi}{6}t) = \frac{-2}{3.5}$

Find related acute angle

$$\frac{\pi}{6}t = \cos^{-1}\left(\frac{2}{3.5}\right)$$



$$\frac{\pi}{6}t = 0.96$$

Our values are $\pi - 0.96 = 2.18$

$\pi + 0.96 = 4.10$

$$\frac{\frac{\cancel{\pi}}{\cancel{6}}t}{\frac{\cancel{\pi}}{\cancel{6}}} = \frac{2.18}{\frac{\pi}{6}} \qquad \frac{\frac{\cancel{\pi}}{\cancel{6}}t}{\frac{\cancel{\pi}}{\cancel{6}}} = \frac{4.10}{\frac{\pi}{6}}$$

$\times 60$ $\qquad$ $\times 60$

$t = 4.16$ $\qquad$ $t = 7.83$

$= 4{:}10\text{am}$ $\qquad$ $= 7{:}50\text{am}$

Other times, add 12 hours

$4{:}10 + 12{:}00 = 16{:}10$ or $4{:}10\text{pm}$

$7{:}50\text{am} + 12{:}00 = 7{:}50\text{pm}$

She could leave at 7:50am and come back at 4:10pm.

Action

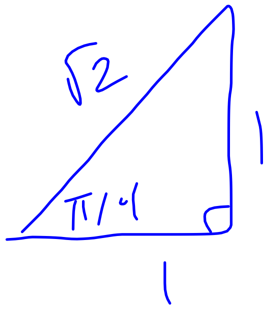
Example 4: Solve 2sinxcosx = cos2x for x on 0 ≤ x ≤ 2π.

$$2\sin x\cos x = \cos 2x$$

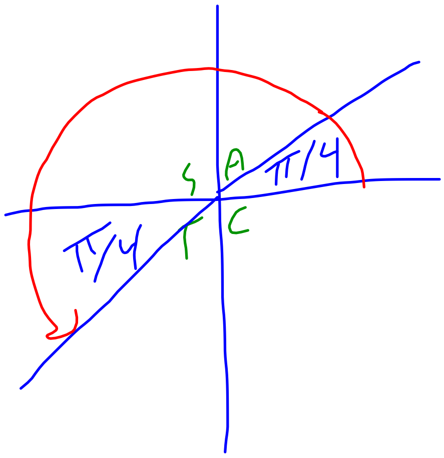
$$\frac{\sin 2x}{\cos 2x} = \frac{\cancel{\cos 2x}}{\cancel{\cos 2x}}$$

$$\frac{\sin 2x}{\cos 2x} = 1$$

$$\tan 2x = 1$$



$$2x = \tan^{-1}(1)$$



$$2x = \frac{\pi}{4}$$

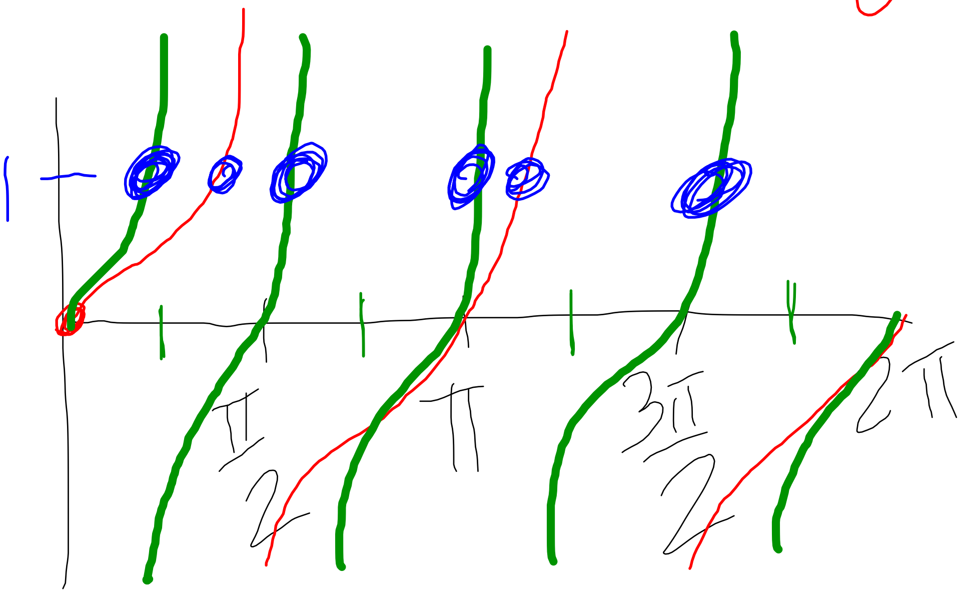
or

$$2x = \pi + \frac{\pi}{4}$$

$$= \frac{5\pi}{4}$$

$2x = \frac{\pi}{4}$ $2x = \frac{5\pi}{4}$

$x = \frac{\pi}{8}$ $x = \frac{5\pi}{8}$



Period of $\tan x = \pi$

$\tan x = 1$ in 2 solutions

Period of $\tan 2x = \frac{\pi}{2}$

$\tan 2x = 1$ has 4 solutions

To find the other solutions, add $\frac{\pi}{2}$ (the period)

$$\frac{\pi}{6} + \frac{\pi}{2} = \frac{5\pi}{6}$$ already known!

$$\frac{5\pi}{6} + \frac{\pi}{2} = \frac{9\pi}{6}$$

We need 1 more!

$$\frac{9\pi}{6} + \frac{\pi}{2} = \frac{13\pi}{6}$$

Solutions are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{9\pi}{6}, \frac{13\pi}{6}$

$$\frac{13\pi}{6} + \frac{\pi}{2} = \frac{17\pi}{6}$$ (more than 2π not a solution)

Pg. 426

3, 6, 8, 9, 10, 12