Learning Goal: I will solve quadratic trig equations.

Minds On: Solving for x.

Action: Solving Quadratic Equations - Note

Consolidation: Practice? Exit Question?

Minds On

Solve for x.

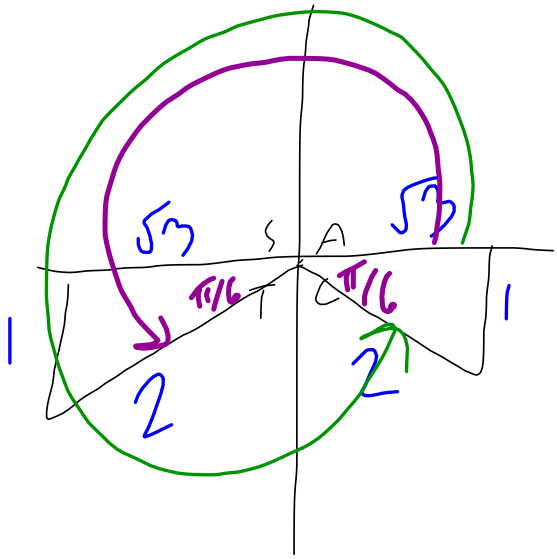
Please try this.

$2 \csc x + 17 = 15 + \csc x$

$-\csc x \quad -17 \quad -17 \quad -\csc x$

$\csc x = -2$

$\sin x = \frac{-1}{2}$



$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

Minds On

Solve for x.

$3x^2 - 2 = 10$

$+2 \quad +2$

$3x^2 = 12$

$x^2 = 4$

$x = \pm 2$

$x^2 + x = 6$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3, 2$

$6x^2 - 4x - 2 = 0$

$2(3x^2 - 2x - 1) = 0$

$2(3x^2 - 3x + 1x - 1) = 0$

$2(3x(x-1) + 1(x-1)) = 0$

$(3x+1)(x-1) = 0$

$x = -\frac{1}{3} \quad x = 1$

$3x^2 + 2x - 4 = 0$

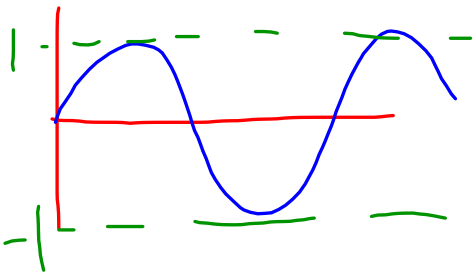
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Minds On

Solve for x.

$\sin x = 1.2$

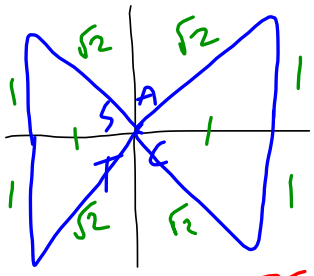
impossible



$\tan^2 x = 1$

$\sqrt{(\tan x)^2} = \sqrt{1}$

$\tan x = \pm 1$



$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Action

Solving Quadratic Trigonometric Equations

Example 1: Solve each equation for x in the interval $0 \leq x \leq 2\pi$.

a) $\sin^2 x - \sin x = 2$

$\sin^2 x - \sin x - 2 = 0$

$(\sin x - 2)(\sin x + 1) = 0$

$\sin x = 2 \quad \sin x = -1$

Optional

Let $\sin x = a$

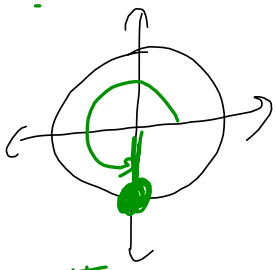
$a^2 - a - 2 = 0$

$(a-2)(a+1) = 0$

$a = 2 \quad a = -1$

~~$\sin x = 2$~~ impossible.

$\sin x = -1$



$x = \frac{3\pi}{2}$

b) $2\sin^2 x - 3\sin x + 1 = 0$

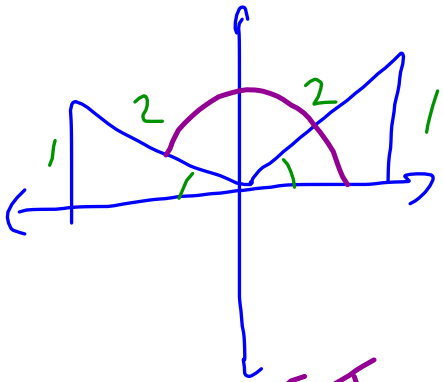
$2\sin^2 x - 2\sin x - 1\sin x + 1 = 0$

$2\sin x(\sin x - 1) - 1(\sin x - 1) = 0$

$(2\sin x - 1)(\sin x - 1) = 0$

$2\sin x - 1 = 0$

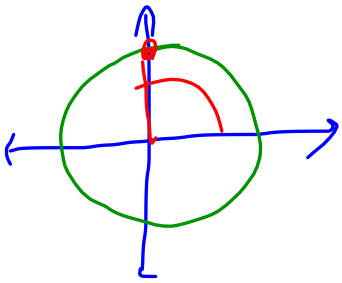
$\sin x = \frac{1}{2}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x - 1 = 0$

$\sin x = 1$



$x = \frac{\pi}{2}$

Example 2: For each equation, use a trigonometric identity to create a quadratic equation. Then solve the equation for x in the interval $[0, 2\pi]$.

a) $2\sec^2 x - 3 + \tan x = 0$

* $1+\tan^2\theta = \sec^2\theta$

$2(1+\tan^2 x) - 3 + \tan x = 0$

$2 + 2\tan^2 x - 3 + \tan x = 0$

$2\tan^2 x + \tan x - 1 = 0$

$2\tan^2 x + 2\tan x - \tan x - 1 = 0$

$2\tan x(\tan x + 1) - 1(\tan x + 1) = 0$

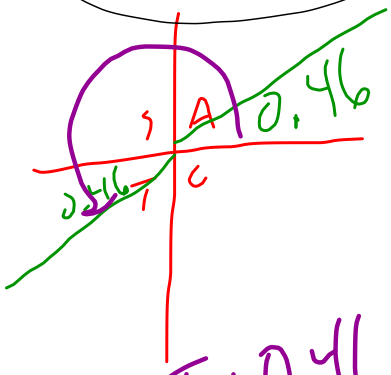
$(2\tan x - 1)(\tan x + 1) = 0$

$\tan x = \frac{1}{2}$ $\tan x = -1$

$\tan x = \frac{1}{2}$

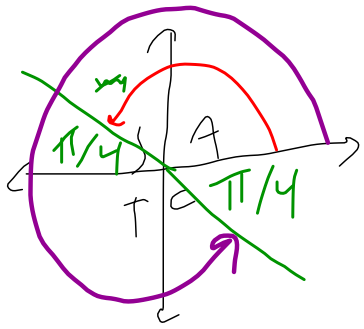
$x = \tan^{-1}\left(\frac{1}{2}\right)$

$x = 0.46$



$x = \pi + 0.46 = 3.60$

$\tan x = -1$



$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$\therefore x = 0.46, 3.60, \frac{3\pi}{4}, \frac{7\pi}{4}$

b) $3\sin x + 3\cos 2x = 2$

$\cos 2\theta = 1 - 2\sin^2\theta$

$3\sin x + 3(1 - 2\sin^2 x) - 2 = 0$

$3\sin x + 3 - 6\sin^2 x - 2 = 0$

$-6\sin^2 x + 3\sin x + 1 = 0$

$$\sin x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\sin x = \frac{-3 \pm \sqrt{3^2 - 4(-6)(1)}}{2(-6)}$$

$\sin x = -0.23 \quad \sin x = 0.73$

$\sin x = -0.23$

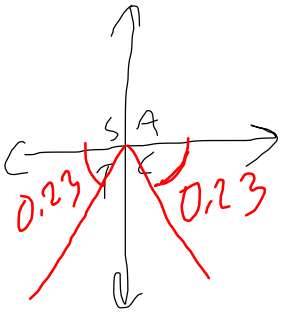
$x = \sin^{-1}(-0.23)$

find r.a.

$x = \sin^{-1}(0.23)$

$x = 0.23$

* sin x was negative



$x = \pi + 0.23$

$x = 3.37$

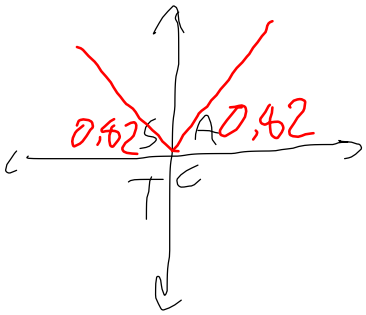
$x = 2\pi - 0.23$

$x = 6.05$

$\sin x = 0.73$

$x = \sin^{-1}(0.73)$

$x = 0.82$

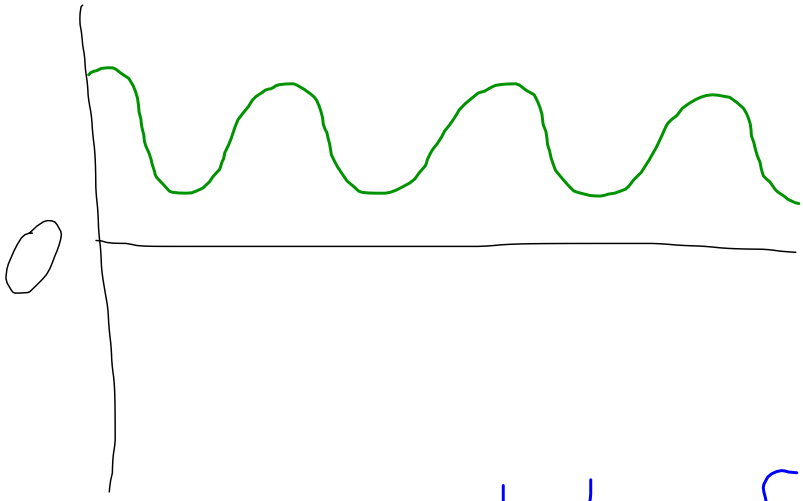


$x = 0.82$

$x = \pi - 0.82$

$x = 2.32$

*It is possible to have no solutions.



If we were looking for zeros, we wouldn't find any

Consolidation

Practice

Pg. 436

4, 7, 8, 12, 13