

Learning Goal: I will be write and interpret logarithmic equations.

Minds On: Zombie Apocalypse and Population Growth

Action: Logarithmic Investigation

Consolidation: Evaluating Logs

Unit 7

Exponential and Logarithmic Functions

Minds On

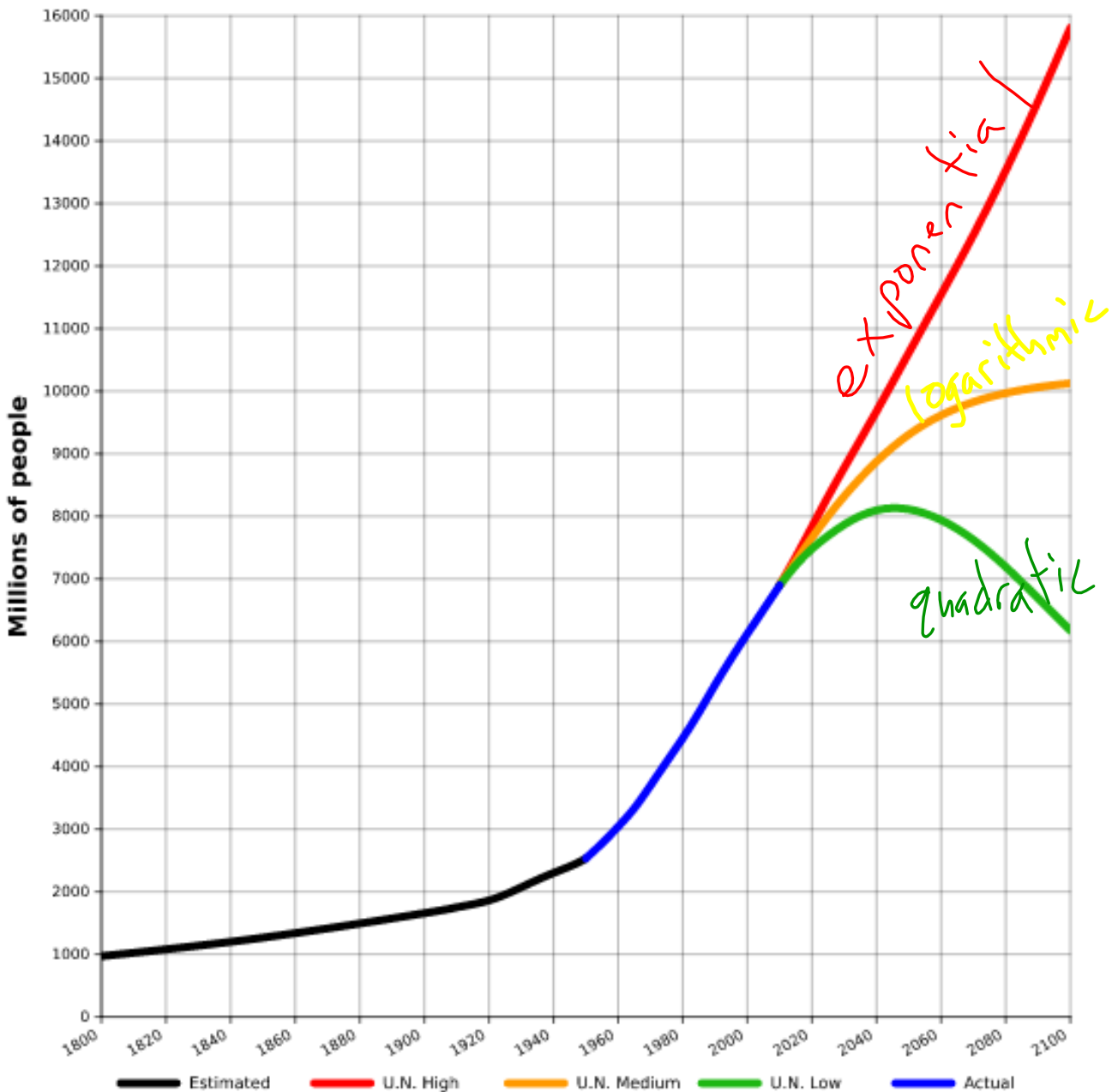
Zombie Apocalypse

On a whiteboard, draw a rough sketch of what you think the spread of a zombie infection might look like over time.



Minds On

World Population



Action

What's the Inverse?

Determine the inverse of each equation, then graph the original function and the inverse.

$$1. y = 3x + 1$$

$$x = 3y + 1$$

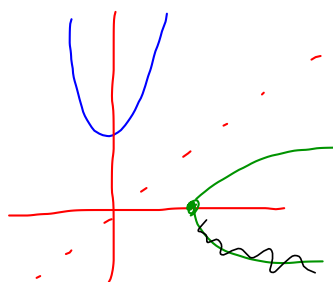
$$y = \frac{x-1}{3} \text{ or } y = \frac{1}{3}x - \frac{1}{3}$$

switch x and y
solve for y

$$2. y = x^2 + 6$$

$$x = y^2 + 6$$

$$y = \sqrt{x-6}$$



$$3. y = 2^x$$

$$x = 2^y$$

Action

Properties of Inverse Functions

The Equations

- switch x & y
- solve for y

The Graphs

- Reflect the graph across $y=x$

The Tables

(x, y) becomes (y, x)
switch x and y values

Action

Desmos Investigation

Desmos Investigation

Graph the function $f(x) = 2^x$ in Desmos.

Complete the table of values below:

x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8
4	16

1. What is the domain of this function?

$$\{x \in \mathbb{R}\}$$

2. What is the range of this function?

$$\{y \in \mathbb{R} \mid y > 0\}$$

Interchange x and y in the equation of f to get the equation of the inverse equation $f^{-1}(x)$.

Graph the inverse equation on the same axes.

How do the two graphs compare?

$x=2^y$ is $y=2^x$ reflected across $x=y / y=x$

Create a table of values for this inverse function, using "nice" values. You may need to move around to find nice points.

x	y
1	0
2	1
4	2
8	3
16	4

1. What is the domain of this inverse function?

$$\{x \in \mathbb{R} \mid x > 0\}$$

2. What is the range of this inverse function?

$$\{y \in \mathbb{R}\}$$

3. How do the points of $f(x)$ seem to relate to those of $f^{-1}(x)$?

x & y are switched

4. Now graph the function $g(x) = \log_2 x$. What do you notice?

Same as $x=2^y$

5. Using each point you found in your last table of values, replace the x and y in the statement $y = \log_2 x$ with the x and y values from your points.

$$\begin{aligned} 0 &= \log_2 1 & 2 &= \log_2 4 & 4 &= \log_2 16 \\ 1 &= \log_2 2 & 3 &= \log_2 8 \end{aligned}$$

6. Based on what you did in #5, what meaning does the expression $\log_a x$ have?

$\log_a x$ = the exponent that must be applied to base a to get the value of x

$$\begin{array}{l} y = a^x \\ \text{inverse} \\ x = a^y \end{array} \rightarrow y = \log_a x$$

7. Remove all other graphs. Now graph $y = a^x$ and $y = \log_a x$.
 Set a slider for a between 0 and 10 with a step of 0.1.
 Complete the table below to describe the behaviour of $y = a^x$ and $y = \log_a x$.

	$y = a^x$	$y = \log_a x$
$a > 1$	increasing	increasing
$0 < a < 1$	decreasing	decreasing
Domain	$\{x \in \mathbb{R}\}$	$\{x \in \mathbb{R} \mid x > 0\}$
Range	$\{y \in \mathbb{R} \mid y > 0\}$	$\{y \in \mathbb{R}\}$
Asymptotes	horizontal: $y = 0$	vertical: $x = 0$
Intercepts	y-intercept: $y = 1$	x-intercept: $x = 1$

Consolidation

Evaluating Logs

$\log_a x$ is the exponent on base a to get x

Evaluate each logarithm below:

$$\log_3 9 = 2$$

$$\log_2 32 = 5$$

$$\log_2 1 = 0$$

$$x^{-1} = \frac{1}{x}$$

$$\log_6 \frac{1}{6} = -1$$

$$\log_4 \frac{1}{16} = -2$$

$$\log_{25} 5 = \frac{1}{2}$$

$$\sqrt{x} = x^{\frac{1}{2}} \quad | \quad \sqrt[3]{x} = x^{\frac{1}{3}} \quad | \quad \sqrt[3]{x^3} = x^{\frac{3}{3}} \quad | \quad \sqrt[m]{x^m} = x^{\frac{m}{m}}$$

$$\log_{16} 2 = \frac{1}{4}$$

$$\log_7 \sqrt{7} = \frac{1}{2}$$

$$\log_2(-4)$$

D.N.E.

Consolidation

Practice

Pg. 451

4 - 6, 9 - 11