**Learning Goal:** I will solve exponential equations.

**Minds On:** 1. Graphs of exponential functions in real life
2. Whiteboards - solve it!

**Action:** Solving Exponential Equations

**Consolidation:** Exit Question - whiteboards or exit cards...depends on time

Today, RAFT for the first bit, then we have the survey, for real!

We will start the lesson after the survey, so RAFT will be cut short.

I'm sorry.

## Minds On

# What's the Equation?

$y = a \cdot b^x$

You start with $M_0$ dollars in the bank, the amount of money you have in the bank:

a. Doubles every year.

$M(t) = M_0 \cdot 2^t$

b. Increases by 75% every year.

$M(t) = M_0 \cdot 1.75^t$

c. Increases by 80% every 3 years.

$M(t) = M_0 \cdot 1.80^{t/3}$

$3 \rightarrow 1$
$6 \rightarrow 2$
$9 \rightarrow 3$

d. Decreases by 15% every 7 years.

$M(t) = M_0 \cdot (0.85)^{t/7}$

## Minds On

# Half-Life

All radioactive substances decrease in mass over time.

What is the general exponential equation for half life?

$$M(t) = M_0 \cdot \left(\frac{1}{2}\right)^{t/h}$$

**Example**

The half life of cesium-137, a radioactive substance released by the Chernobyl accident, is 30 years. 1986

If 4 kg of the substance was released in the accident,

a. How much remains today? 30yrs $M(30) = 4 \cdot \left(\frac{1}{2}\right)^{30/30}$ $M(30) = 2$

b. How much will remain in 2030? $M(44) = 4 \cdot \left(\frac{1}{2}\right)^{44/30}$ $M(44) =$

c. When will there be less than 1 kg remaining?

d. When will there be less than 200 g remaining?

$3^x = 15$

$1 = 4 \cdot \frac{1}{2}^{t/30}$

$0.25 = \frac{1}{2}^{t/30}$

# Action Solving Exponential Equations

Example 1: Different strategies to solve an exponential equation

All radioactive substances decrease in mass over time. Kristen works in a laboratory that uses radioactive substances. T he laboratory received a shipment of 200 g of radioactive radon, and 16 days later, 12.5 g of the radon remained. What is the half-life of radon?

*Solution A*

$$M(t) = M_0 \cdot \frac{1}{2}^{t/h}$$

$$\frac{12.5}{200} = \frac{\cancel{200} \cdot \frac{1}{2}^{16/h}}{\cancel{200}}$$

$$\frac{12.5}{200} = \frac{1}{2}^{\frac{16}{h}}$$

$$\frac{1}{16} = \frac{1}{2}^{\frac{16}{h}}$$

$$\frac{1}{2}^{4} = \frac{1}{2}^{16/h}$$

$$4 = \frac{16}{h}$$

$$h = 4$$

# Action

*Solution B*

$$M(t) = M_0 \times \frac{1}{2}^{t/h}$$

$$\frac{12.5}{200} = \frac{\cancel{200} \times \frac{1}{2}^{16/h}}{\cancel{200}}$$

$$\log 0.0625 = \log 0.5^{16/h}$$

$$\log 0.0625 = \frac{16}{h} \log 0.5$$

$$\frac{16 \log 0.5}{h}$$

$$\log 0.0625 = \frac{16 \log 0.5}{h}$$

$$h = \frac{16 \cdot \log 0.5}{\log 0.0625}$$

$$h = 4$$

# Action

*Solution C*

Graph both sides.
Find POI

# Action

<u>Example 2: Using Logs to solve a problem</u>

An investment of \$2500 (P) grows at a rate of 4.8% (→ 0.048) per year, compounded annually.

How long will it take for the investment to be worth \$4000? → A

Recall that the formula for compound interest is $A = P(1+i)^n$.

$$\frac{4000}{2500} = \frac{2500(1.048)^n}{2500}$$

$$\log 1.6 = \log 1.048^n$$

$$\frac{\log 1.6}{\log 1.048} = \frac{n \cancel{\log 1.048}}{\cancel{\log 1.048}}$$

$$n = 10.025$$

# Action

Example 3: Exponentials with more than one power

Solve $2^{x+2} - 2^x = 24$

$$2^{x+2} - 2^x = 24$$

$$2^x(2^2 - 1) = 24$$

$$\frac{2^x(3)}{3} = \frac{24}{3}$$

$$2^x = 8$$

$$x = 3$$

$$\log 2^x = \log 8$$

$$x \log 2 = \log 8$$

$$x = \frac{\log 8}{\log 2}$$

# Action

Example 4: When the exponents have different bases

Solve $2^{x+1} = 3^{x-1}$ to three decimal places.

$$\log 2^{x+1} = \log 3^{x-1}$$

$$(x+1)\log 2 = (x-1)\log 3$$

$$x\log 2 + \log 2 = x\log 3 - \log 3$$

$$\underbrace{x\log 2 - x\log 3} = -\log 3 - \log 2$$

$$\frac{x\cancel{(\log 2 - \log 3)}}{\cancel{\log 2 - \log 3}} = \frac{-\log 3 - \log 2}{\log 2 - \log 3}$$

$$x = 4.419$$

$2^x = 8$

$x = 3$

$\log 2^x = \log 8$

$x \log 2 = \log 8$

$x = \frac{\log 8}{\log 2}$

$2^x = 8$

$x = \frac{\log 8}{\log 2}$

$2^x = 8$

$\log_2 8 = x$

$x = \frac{\log 8}{\log 2}$

$\log_3 7 = x$

$x = 1.77$

$$y = \log_a x$$

$$y = \frac{\log x}{\log a}$$

## Consolidation

Solve: $9^{2x + 1} = 81(27^{x})$

## Consolidation

# Practice

Pg. 485

1 - 3 (a few from each)

4, 5, 8, 10, 11