Learning Goal: I will use exponential and logarithmic functions to solve problems involving growth and decay.

Minds On: Quick as a Bunny

Action: Solving Logarithmic Equations

Consolidation: Exit Questions

Action

Solving Problems with Exponential and Logarithmic Functions

Example 1:

In chemistry, the pH (the measure of acidity or alkalinity of a substance) is based on a logarithmic scale. A logarithmic scale uses powers of 10 to compare numbers that vary greatly in size. For example, very small and very large concentrations of the hydrogen ion in a solution influence its classification as either a base or an acid. A difference of one pH unit represents a tenfold (10 times) change in the concentration of hydrogen ions in the solution. For example, the acidity of a sample with a pH of 5 is 10 times greater than the acidity of a sample with a pH of 6. A difference of 2 units, from 6 to 4, would mean that the acidity is 100 times greater, and so on. Anything with a pH less than 7 is considered acidic; greater than 7 is considered alkaline, and anything equal to 7 is considered to be neutral. The relationship between pH and hydrogen ion concentration is given by the formula **$pH = -\log [H^+]$**, where $[H^+]$ is the concentration of hydrogen ions in moles per litre (mol/L).

a) Calculate the pH if the concentration of hydrogen ions is 0.0001 mol/L.

$pH = -\log[H^+]$ $\quad$ $pH = 4$

$pH = -\log[0.0001]$

b) The pH of lemon juice is 2. Calculate the hydrogen ion concentration.

$\underline{2}_{-1} = \underline{-\log[H^+]}_{-1}$ $\quad$ $-2 = \log[H^+]$

$[H^+] = 10^{-2} = 0.01$

c) If the hydrogen ion concentration is a measure of the strength of an acid, how much stronger is an acid with pH 1.6 than an acid with pH 2.5.

Find H^+ for each

$1.6 = -\log[H^+]$ $\quad$ $2.5 = -\log[H^+]$

$-1.6 = \log[H^+]$ $\quad$ $-2.5 = \log[H^+]$

$[H^+] = 10^{-1.6}$ $\quad$ $[H^+] = 10^{-2.5}$

$$\frac{10^{-1.6}}{10^{-2.5}} \doteq 7.9$$

it is 7.9 times stronger

Example 2:

The Richter magnitude scale uses logarithms to compare intensities of earthquakes.

True Intensity		Richter Scale Magnitude
10^1		$\log_{10}10^1 = 1$
~~10^2~~	10^4	$\log_{10}10^4 = 4$
~~10^3~~	$10^{5.8}$	$\log_{10}10^{5.8} = 5.8$

An earthquake of magnitude 2 is actually 10 times more intense than an earthquake of magnitude 1. The difference between the magnitudes of two earthquakes can be used to determine the difference in intensity. If the average earthquake measures 4.5 on the Richter scale, how much more intense is an earthquake that measures 8?

intensity of $R=4.5$ $I = 10^{4.5}$

intensity of $R=8$ $I = 10^8$

$$\frac{10^8}{10^{4.5}} = 10^{3.5}$$

$= 3{,}162$ times more intense

Example 3:

97.8% remains

Blue jeans fade when washed due to the loss of blue dye from the fabric. If each washing removes about 2.2% of the original dye from the fabric, how many washings are required to give a pair of jeans a well-worn look? (For a well-worn look, jeans should contain, at most, 30% of the original dye.)

"the look"

$$y = a \cdot b^x$$

washings

$100\% = 1$

$$L(w) = 1 \cdot (1 - 0.022)^w$$

$$0.3 = 1 \times 0.978^w$$

$$0.3 = 0.978^w$$

$$\log 0.3 = \log 0.978^w$$

$$\frac{\log 0.3}{\log 0.978} = \frac{w \cdot \log 0.978}{\log 0.978}$$

$w = 54$ washings

Example 4:

The dynamic range of human hearing and sound intensity spans from 10^{-12} W/m^2 to about 10 W/m^2. The highest sound intensity that can be heard is 10 000 000 000 000 times as loud as the quietest. This span of sound intensity is impractical for normal use. A more convenient way to express loudness is a relative logarithmic scale, with the lowest sound that can be heard by the human ear, $I_0 = 10^{-12}$ W/m^2, given the measure of loudness of 0 dB.

The formula that is used to measure sound is $L = 10\log\left(\frac{I}{I_0}\right)$, where L is the loudness measured in decibels, I is the intensity of the sound being measured, and I_0 is the intensity of sound at the threshold of hearing.

a. The loudness of a rock concert is 120 dB, and the loudness of a subway is 90 dB. How many times more intense is the sound of a rock concert than the sound of a subway?

$$\frac{I_{RC}}{I_{SW}}$$

$$L = 10\log\left(\frac{I}{I_o}\right)$$

Rock Concert

$$\frac{120}{10} = \frac{10\log\left(\frac{I_{RC}}{10^{-12}}\right)}{10}$$

$$12 = \log_{10}\left(\frac{I_{RC}}{10^{-12}}\right)$$

$$10^{12} = \frac{I_{RC}}{10^{-12}}$$

$$I_{RC} = 10^{-12} \times 10^{12}$$

$$I_{RC} = 10^{0}$$

$$I_{RC} = 1$$

Subway

$$90 = 10\log\left(\frac{I_{SW}}{10^{-12}}\right)$$

$$9 = \log_{10}\left(\frac{I_{SW}}{10^{-12}}\right)$$

$$10^{9} = \frac{I_{SW}}{10^{-12}}$$

$$I_{SW} = 10^{9} \times 10^{-12}$$

$$I_{SW} = 10^{-3}$$

$$I_{SW} = \frac{1}{1000}$$

$$\frac{I_{RC}}{I_{SW}} = \frac{10^{0}}{10^{-3}}$$

$$= 10^{0 - -3}$$

$$= 10^{3}$$

$= 1000$ times more intense

b. The sound of a power mower is 5 times more intense than that of a snowmobile. If the loudness of a power ~~motor~~ mower is 107 dB, what is the loudness of a snow mobile?

$$L = 10\log\left(\frac{I}{10^{-12}}\right)$$

1. Find intensity of power mower

$$107 = 10\log\left(\frac{I}{10^{-12}}\right)$$

$$10.7 = \log\left(\frac{I}{10^{-12}}\right)$$

$$10^{10.7} = \frac{I}{10^{-12}}$$

$$I = 10^{10.7} \times 10^{-12}$$

$$I = 10^{-1.3}$$

2. Intensity of snowmobile $= \dfrac{10^{-1.3}}{5}$

3. Find loudness of snowmobile

$$L = 10\log\left(\frac{\frac{10^{-1.3}}{5}}{10^{-12}}\right)$$

$$L = 10\log\left(\frac{10^{-1.3}}{5 \times 10^{-12}}\right)$$

$$L = 10\log\left(\frac{10^{-1.3-(-12)}}{5}\right)$$

$$L = 10\log\left(\frac{10^{10.7}}{5}\right)$$

$$L = 100\text{ dB}$$

Consolidation

Practice

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