MHF4U Culminating - Unit 1 Polynomial Functions

Knowledge:
True or False (Correct if False):

1.) For the function $y = 3(-\frac{1}{2}(x+2))^5 + 4$

a) There is a reflection in the x-axis.

False - reflection in the y-axis.

b) It has a horizontal translation 2 units left.

True

c) It has a horizontal stretch by a factor of ½.

False - horizontal stretch by factor of 2

d) An even degree polynomial (degree 6) with a positive coefficient will always extend from quadrant 2 to quadrant 1.

True

e) An odd degree polynomial will always have an odd number of turning points.

False - even # of turning points.

f) Is the following statement true or false: dividend = divisor x quotient + remainder.

True

2.) The Remainder Theorem: When does a polynomial have a factor?

when $f(x)$ is being divided by $x-a$ & the remainder is equal to $f(a)$ ($f(a)$ must equal ∅)

3.) What are the restrictions for using synthetic division?

- must divide by a linear expression.
- that expression must have a leading coefficient of 1

4.) Divide the polynomial $3x^2 + 4x + 6$ by $x - 3$

$$
\begin{array}{r}
3x + 5 \\
x-3 \overline{)\,3x^2 + 4x + 6} \\
3x^2 - 9x \downarrow \\
\hline
5x + 6 \\
5x - 15 \\
\hline
21 \text{ R.}
\end{array}
$$

5.) Use the remainder theorem to determine if the following are factors of the polynomial $6x^3+5x^2-16x-15$.

a.) x+1

$$
\begin{array}{r|rrrr}
-1 & 6 & 5 & -16 & -15 \\
 & \downarrow & -6 & 1 & 15 \\
\hline
 & 6x^2 & -1x & -15 & \emptyset \text{R}
\end{array}
$$

$\therefore$ yes, $x+1$ is a factor of $6x^3+5x^2-16x-15$.

b.) 2x+3

$$
\begin{array}{r}
3x^2 - 2x - 5 \\
2x+3 \overline{)\,6x^3 + 5x^2 - 16x - 15} \\
-\ 6x^3 + 9x^2 \downarrow \\
\hline
-4x^2 - 16x \\
-\ -4x^2 - 6x \\
\hline
-10x - 15 \\
-\ -10x - 15 \\
\hline
\emptyset \text{ R.}
\end{array}
$$

$\therefore$ yes, $2x+3$ is a factor of $6x^3+5x^2-16x-15$.

c.) x-1

$$
\begin{array}{r|rrrr}
1 & 6 & 5 & -16 & -15 \\
  & \downarrow & 6 & 11 & -5 \\
\hline
  & 6x^2 & +11x & -5 & -20\ R
\end{array}
$$

$\therefore$ No, $x-1$ is <u>not</u> a factor of $6x^3+5x^2-16x-15$

6.) Name the following terms when dividing polynomials:

$$
\begin{array}{rl}
 & 26 \leftarrow \text{quotient} \\
\text{divisor} \rightarrow 4 \,\big)\!\! & \overline{107} \leftarrow \text{dividend.} \\
 & -8\downarrow \\
\hline
 & 27 \\
 & -24 \\
\hline
 & 3 \leftarrow \text{remainder}
\end{array}
$$

7.) Identify the transformations for a, k, d & c from the original function f(x) = x^4 to *g(x) = -½ (6 (x - 3))⁴ + 8.*

a = -1/2 : verticle compression by factor of 2 & the negative is a reflection in the x-axis.

K = 6 : horizontal compression by a factor of 6.

d = 3 : horizontal shift 3 units right.

c = 8 : verticle shift 8 units up.

Application:

1.) Given the table of values below for a cubic function, determine the new coordinates of the points after the function undergoes the transformations in the function
$g(x) = -2\,(\,2\,(\,x + 3\,)\,)^3 + 4$.

x	y
-2	-4
-1	-2
0	0
1	2
2	4

$\frac{x}{2}+3$	$-2y+4$
2	12
$\frac{5}{2}$	6
3	4
$\frac{7}{2}$	0
4	-4

2.) Factor the difference of cubes below:

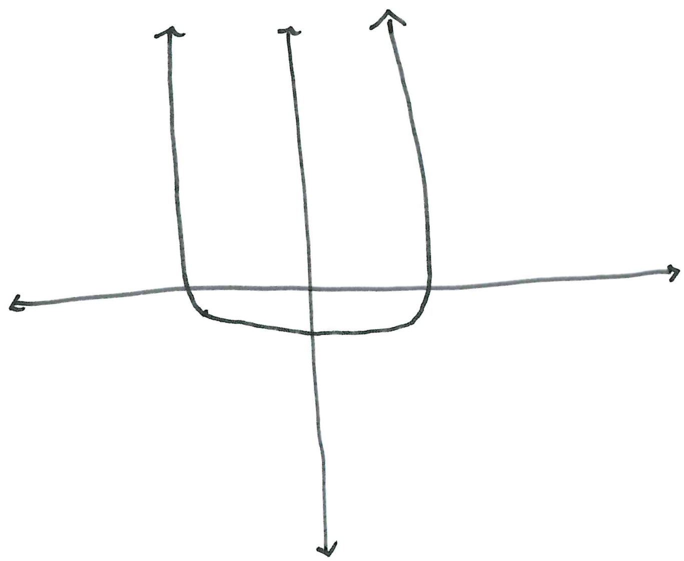
a.) $f(x) = x^3 - 125$

$$= (x-5)(x^2+5x+25)$$

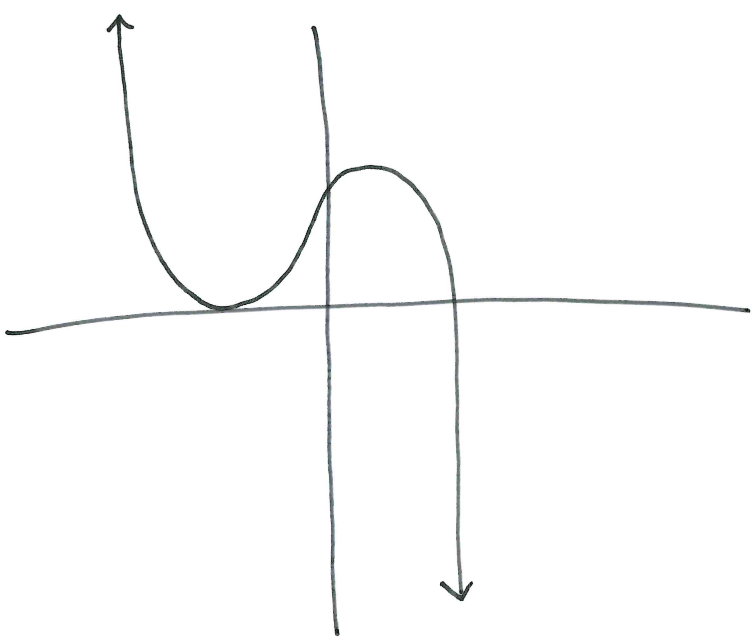
b) $f(x) = 125x^3 - 27y^3$

$$= (5x-3y)(25x^2+15xy+9y^2)$$

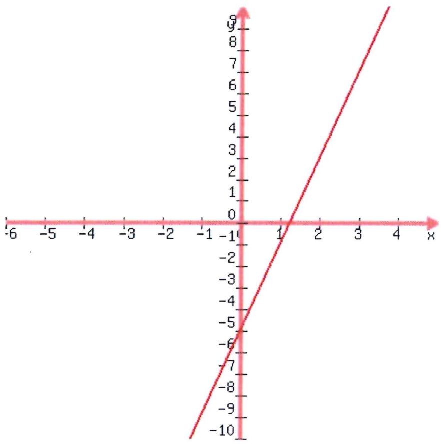
3. a) Sketch a graph of a function with a positive leading coefficient, a degree of 4, 2 zeros, and one turning point.



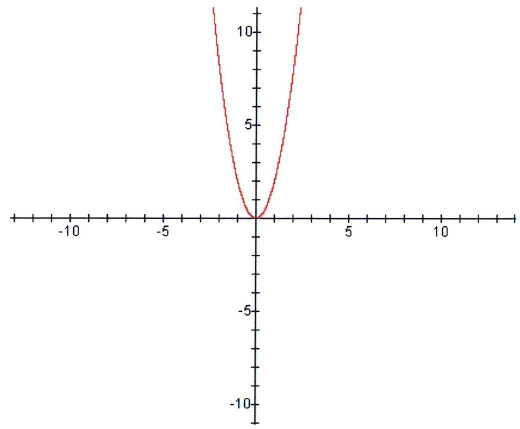
b) Sketch a graph of a function with a negative leading coefficient, a degree of 3, 2 zeros, and 2 turning points.



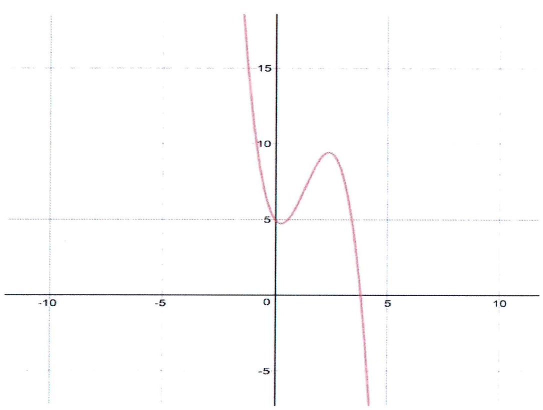
4. Match the functions to their graphs by writing the correct equation underneath. 2 equations do not belong.



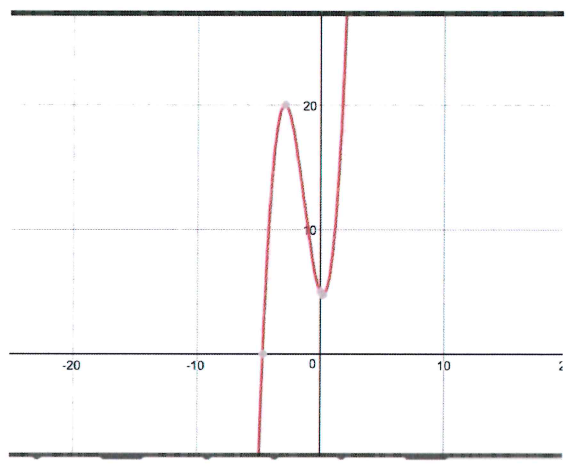
$4x-1$



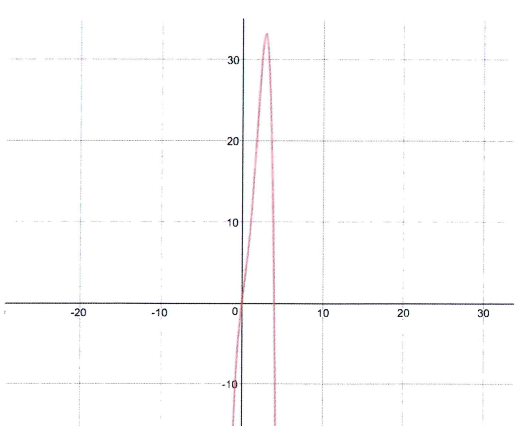
$2x^2$



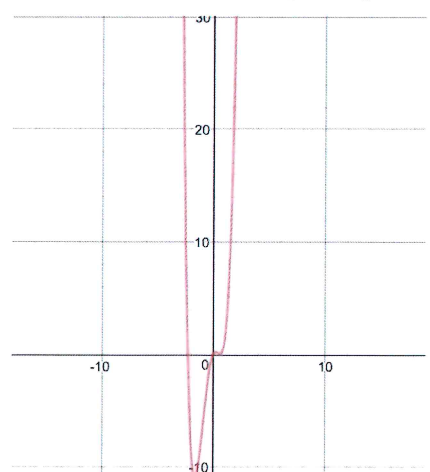
$-x^3+4x^2-2x+5$



x^3+4x^2-2x+5



$-x^4+4x^3-2x^2+8x$



$2x^2+2x^3-5x^2+2x$

A. $f(x) = x^3 + 4x - 2x + 5$ B. $f(x) = -2x - 1$ C. $f(x) = -3x^2$ D. $f(x) = 4x - 1$
E. $f(x) = 2x^4 + 2x^3 - 5x^2 + 2x$ F. $f(x) = -x^3 + 4x^2 - 2x + 5$ G. $f(x) = 2x^2$

TIPS:

1.) Determine the equation of the graph to the right.

zeros @ $-5, -1, 2$

y int. $= -2$

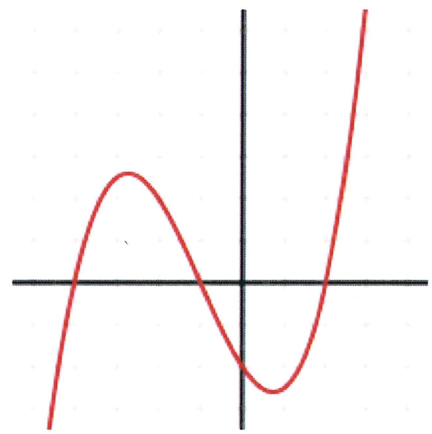
$f(x) = a(x+5)(x+1)(x-2)$

sub in y int. $(0,-2)$

$-2 = a(0+5)(0+1)(0-2)$

$-2 = -10a$

$a = \frac{1}{5}$



$\therefore$ the eq'n is $f(x) = \frac{1}{5}(x+5)(x+1)(x-2)$

2.) John divided $x^3 - 4x^2 + 2x + 5$ by a polynomial. His answer was $x^2 - 2x - 2$ with a remainder of 1. What polynomial did John divide by?

dividend = quotient × divisor + remainder

$\therefore$ divisor $= \dfrac{\text{dividend} - \text{remainder}}{\text{quotient}}$

plug in values

divisor $= \dfrac{x^3 - 4x^2 + 2x + 5 - 1}{x^2 - 2x - 2}$

$= x^2 - 2x - 2 \overline{\smash{)}\, x^3 - 4x^2 + 2x + 4}$ $\quad \boxed{x - 2}$

$\therefore$ John divided by $x - 2$

3.)Solve the following polynomial to find the values of b and c if it has zeros at (x - 2) and (x + 3).
$f(x) = 3x^3 + bx^2 + cx + 2$

Look at the zeros 2, -3

$0 = 3(2)^3 + b(2)^2 + c(2) + 2$
$= 24 + 4b + 2c + 2$
$= 26 + 4b + 2c$
$\boxed{c = -2b - 13}$ ①

$0 = 3(-3)^3 + b(-3)^2 + c(-3) + 2$
$= -81 + 9b - 3c + 2$
$\boxed{9b - 3c = 79}$ ②

sub ① into ②
$9b - 3(-2b - 13) = 79$
$15b = 40$
$\boxed{b = \frac{8}{3}}$

sub b into ①
$c = -2(\frac{8}{3}) - 13$
$c = -\frac{55}{3}$

$\therefore b = \frac{8}{3}$ or 2.6667
$c = \frac{-55}{3}$ or $-18.\overline{3}$

4.) When $2x^3 - ax^2 + bx - 2$ is divided by x + 1 the remainder is -12 and x - 2 is a factor. Determine the values of a and b.

$f(x) = 2x^3 - ax^2 + bx - 2$
look @ f(-1)
$f(-1) = 2(-1)^3 - a(-1)^2 + b(-1) - 2$
remainder
$-12 = 2(-1)^3 - a(-1)^2 + b(-1) - 2$
$= -2 - a - b - 2$
$\boxed{a = 8 - b}$ ①

Look @ f(2)
$0 = 2(2)^3 - a(2)^2 + b(2) - 2$
$0 = 16 - 4a + 2b - 2$
$0 = -4a + 2b + 14$
sub in ①

$0 = -4(8 - b) + 2b + 14$
$18 = 6b$
$\boxed{b = 3}$
sub into ① $\rightarrow$ $a = 8 - 3$
$\boxed{a = 5}$

$\therefore a = 5, b = 3$

Communication:

1.) Write an equation for the following transformations of the original function $f(x) = x^4$

- Vertical stretch by a factor of 2.
- Reflection in the y-axis.
- Translated 6 units right.
- Translated 1 unit down.

$$f(x) = 2(-(x-6))^4 - 1$$

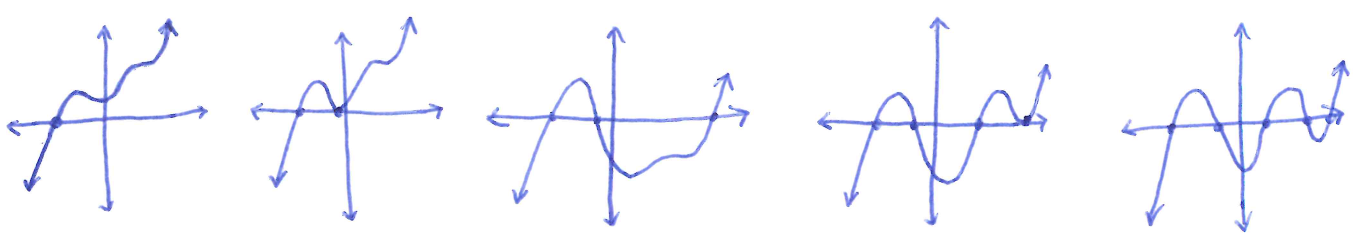
2.) Determine the degree of the function from the following chart:

x	y
0	0
1	1
2	8
3	27
4	64
5	125

> 1 > 7 > 19 > 37 > 61
> 6 > 12 > 18 > 24
> 6 > 6 > 6

∴ it's highest degree is cubic (x^3).

3.) How many zeros can a quintic function have? Use diagrams to support your answer.



1 zero. 2 zeros. 3 zeros 4 zeros. 5 zeros.

because its an odd degree function (5) it goes from quadrant 3 to 1 it must have at least 1 zero but can have up to 5 zeros.

4.) Describe the steps in which you would use to factor a function of degree 5 with a leading coefficient of 1 and a constant of 8.

ex. $x^5 + 4x^4 + 2x^3 + 6x^2 + 8$

1. find a factor by plugging in numbers to make the function = ∅
2. divide using the factor found
3. repeat steps 1 & 2 until you're left w/ a polynomial of degree 2
4. factor remaining function.

5.) In the following equation, $4x^4+6x^2-3x+8$, what is an extra step that is crucial for dividing polynomials?

adding in a place holder for the cubic term.

$\overline{)4x^4 + 0x^3 + 6x^2 - 3x + 8}$