Claudia and Emily

Trigonometric Function Identities

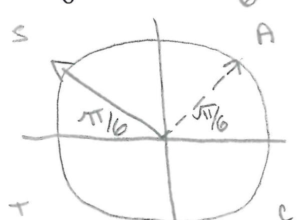
It's the Christmas holiday again, thank goodness, and your parents are wanting to go out and see a show with your aunt and uncle. The only catch is they can't go unless you babysit your younger cousins Lucy and Jake. You almost refuse, but then your aunt offers to pay you a substantial amount of money. Being broke from Christmas, you accept and head over to their house. Your cousin Lucy is the youngest cousin being 5 years old and Jake is already 11. To be productive you work on the math homework you were given for the holiday while they watch tv. Jake keeps coming over and asking you questions about what you are doing. Finally you give in and start showing him what you are working on. It is a practice test for your final exam and the test questions are below..

<u>For exam, practice giving exact answers where possible</u>
Approximate answers can be rounded to 2 decimal places

<u>Part A: Knowledge and Understanding:</u>

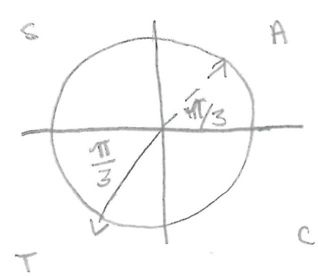
1. Determine an equivalent expressions using the related acute angle

$\cos \frac{5\pi}{6} = -\cos \frac{\pi}{6}$



Cos is (-) in quadrant II so you have to make it (-) in quadrant I too.

$\tan \frac{4\pi}{3} = \tan \frac{\pi}{3}$



2. Use cofunction identities to find an equivalent expression

$\sin\theta = \cos \frac{\pi}{2} - \theta$

$\tan \frac{2\pi}{8} = \cot\left(\frac{\pi}{2} - \frac{2\pi}{8}\right)$

$= \cot\left(\frac{\pi \times 4}{2 \times 4} - \frac{2\pi}{8}\right)$

$= \cot\left(\frac{4\pi}{8} - \frac{2\pi}{8}\right)$

$= \cot\left(\frac{2\pi}{8}\right)$

$= \cot \frac{\pi}{4}$

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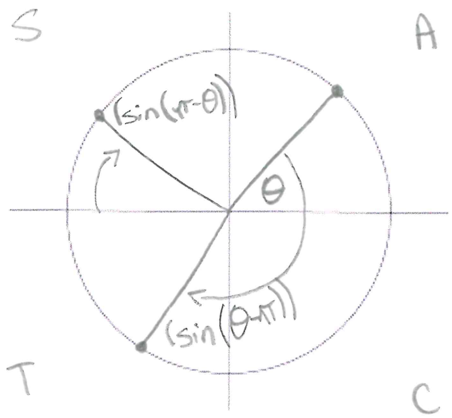
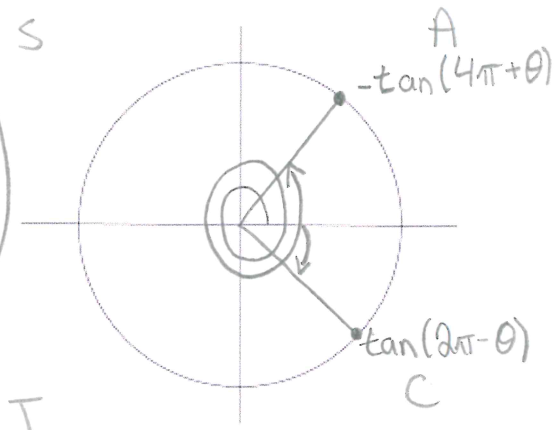
3. Using the unit circle, determine whether or not the following equations are true.

a) $\sin(\pi-\theta) = \sin(\theta-\pi)$



$\sin(\pi-\theta)$ is positive and $\sin(\theta-\pi)$ is negative. $\therefore \sin(\pi-\theta) \neq \sin(\theta-\pi)$

b) $\tan(2\pi-\theta) = -\tan(4\pi+\theta)$



$\tan(2\pi-\theta)$ is negative and $-\tan(4\pi+\theta)$ is negative.

$\therefore \tan(2\pi-\theta) = -\tan(4\pi+\theta)$

4. Express each of the following using two compound angles.

a) $30° = 90° - 60°$ or $\frac{\pi}{2} - \frac{\pi}{3}$

$= 60° - 30°$ or $\frac{\pi}{3} - \frac{\pi}{6}$

b) $\frac{2\pi}{6}$ ($\frac{\pi}{3}$ or $60°$) $= 30° + 30°$ or $\frac{\pi}{6} + \frac{\pi}{6}$

$= 90° - 30°$ or $\frac{\pi}{2} - \frac{\pi}{6}$

5. Simplify each expression.

a) $\sin 60° \cos 30° - \cos 60° \sin 30°$

$= \sin(60° - 30°)$

$= \sin(30°)$

$= \sin\left(\frac{\pi}{6}\right)$

$= \frac{1}{2}$

Formula:

$\sin(a-b) = \sin a \cos b - \cos a \sin b$.

b) $\dfrac{2\tan\frac{3\pi}{4}}{1-\tan^2\frac{3\pi}{4}}$

$= \tan\left(2 \cdot \frac{3\pi}{4}\right)$

$= \tan\left(\frac{2}{1} \cdot \frac{3\pi}{4}\right)$

$= \tan\left(\frac{6\pi}{4}\right)$

Formula:

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

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6. Prove how the following equation isn't an identity.

$1 - sin^2\theta = cos(2\theta)$

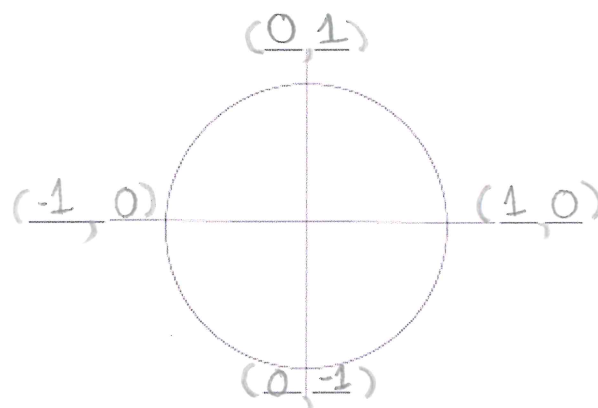
$1 - 2sin^2\theta = cos(2\theta)$

$1 - sin^2\theta = cos^2\theta$

$cos^2\theta \neq cos(2\theta)$

$\therefore$ The equation is not an identity.

7. Label all of the coordinates on the unit circle and fill in below what trig function each represents



X coordinates are cos

Y coordinates are sin

Part B: Application

1.Prove that the following are trigonometric identities

a) $csc^2\theta = cos^2\theta + cos(2\theta) + cos^2\theta + \frac{cos^2\theta}{sin^2\theta}$

RS

$cos^2\theta + cos(2\theta) + cos^2\theta + \frac{cos^2\theta}{sin^2\theta}$

$= sin^2\theta + cos^2\theta + \frac{cos^2\theta}{sin^2\theta}$

$= 1 + \frac{cos^2\theta}{sin^2\theta}$

$= 1 + cot^2\theta$

$= csc^2\theta$

$\therefore LS = RS$ and $csc^2\theta = cos^2\theta + cos(2\theta) + cos^2\theta + \frac{cos^2\theta}{sin^2\theta}$

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b) $\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$

LS

$\tan^2\theta - \sin^2\theta$

$= \frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta$

← factor out $\sin^2\theta$

$= \sin^2\theta\left(\frac{1}{\cos^2\theta} - 1\right)$

$= \sin^2\theta\,(\sec^2\theta - 1)$

$= \sin^2\theta\tan^2\theta$

$\therefore$ LS = RS

$\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$.

c) $\tan 2x - 2\tan 2x \sin^2 x = \sin 2x$

LS

$\tan 2x - 2\tan(2x)\sin^2 x$

$= \tan 2x\,(1 - 2\sin^2 x)$

$= \tan 2x\,(\cos(2x))$

$= \frac{\sin 2x}{\cos 2x} \cdot \frac{\cos 2x}{1}$

$= \sin 2x$

$\therefore$ LS = RS

$\tan 2x - 2\tan(2x)\sin^2 x = \sin 2x$.

d) $\frac{1-(2(1-\cos^2\theta))}{2(\sin\theta\cos\theta)} = \frac{1-\tan^2\theta}{2\tan\theta}$

* cross multiply twice to flip both equations.

$\frac{2(\sin\theta\cos\theta)}{1-(2(1-\cos^2\theta))} = \frac{2\tan\theta}{1-\tan^2\theta}$

$\frac{2(\sin\theta\cos\theta)}{1-(2(1-\cos^2\theta))} = \tan 2\theta$

LS

$\frac{2(\sin\theta\cos\theta)}{1-(2(1-\cos^2\theta))}$

$= \frac{\sin 2\theta}{1-2\sin^2\theta}$

$= \frac{\sin 2\theta}{\cos 2\theta}$

$= \tan 2\theta$.

$\therefore$ LS = RS

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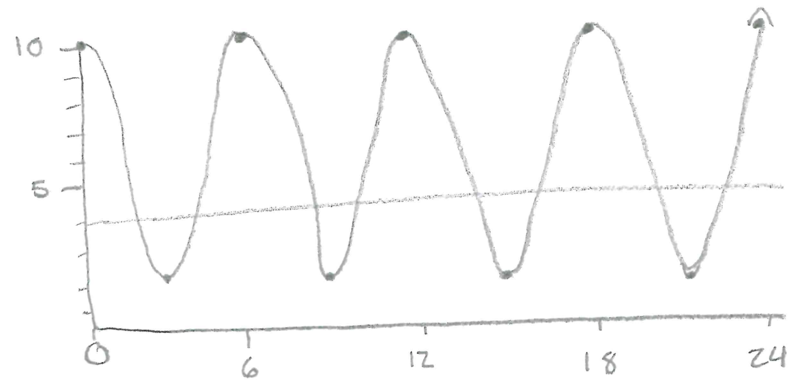
2. Melinda likes to play with electricity. She is a trained professional, don't worry. The electrical current can be expressed by $e(t) = 4\cos(\frac{\pi}{3}t) + 6$ where t represents time in hours $(0 \le t \le 24)$ and e represents amount of electricity passing through a single point measured in kilowatts. If the current reaches 4 kilowatts then she must jump in order to avoid being shocked through the floor of the house she is setting up. How often and at what time must Melinda jump in order to avoid electrocution?



$e(t) = 4\cos(\frac{\pi}{3}t) + 6$

Amplitude = 4

Axis = 6

max = 10

min = 2

period = $\frac{2\pi}{\frac{\pi}{3}} = 6$

$4 = 4\cos(\frac{\pi}{3}t) + 6$
$-6 \qquad -6$

$\frac{-2}{4} = \frac{4\cos(\frac{\pi}{3}t)}{4}$

$-1/2 = \cos(\frac{\pi}{3}t) \rightarrow \cos^{-1} -1/2$

RAA = $\pi/3$

$\pi + \pi/3 = \frac{4\pi}{3}$ $\qquad$ $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$\frac{4\pi}{3} = \frac{\pi}{3}t$ $\qquad$ $\frac{2\pi}{3} = \frac{\pi}{3}t$

$\frac{\frac{4\pi}{3}}{\frac{\pi}{3}} = t$ $\qquad$ $\frac{\frac{2\pi}{3}}{\frac{\pi}{3}} = t$

$\frac{4\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{\pi}} = t$ $\qquad$ $\frac{2\cancel{\pi}}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{\pi}} = t$

$4 = t$ $\qquad$ $2 = t$

$\downarrow$ $\qquad$ $\downarrow$

4, 10, 16, 22, $\qquad$ 2, 8, 14, 20

∴ Melinda must jump at 2am, 4am 8am, 10am, 2pm, 4pm, 8pm, and 10pm.

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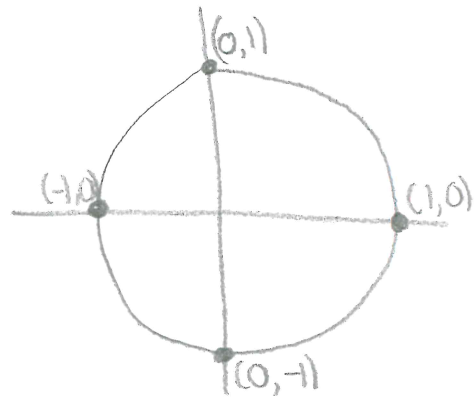
3. Determine the exact value of the following ratios.

a) $\cos\left(\pi + \frac{\pi}{2}\right)$



$= \cos\pi\cos\frac{\pi}{2} - \sin\pi\sin\frac{\pi}{2}$

$= (-1)(0) - (0)(1)$

$= 0$

$\therefore \cos\left(\pi + \frac{\pi}{2}\right) = 0$

Formula: $\cos(a+b) = \cos a\cos b - \sin a\sin b$

b) $\tan 75°$

$\tan 75° = \tan(30° + 45°)$

$= \dfrac{\tan 30° + \tan 45°}{1 - \tan 30° \tan 45°}$

$= \dfrac{\frac{1}{\sqrt{3}} + 1}{1 - \left(\frac{1}{\sqrt{3}}\right)(1)}$

$= \dfrac{\frac{1}{\sqrt{3}} + 1}{1 - \frac{1}{\sqrt{3}}}$

$= \dfrac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}}$ *Not rationalized.

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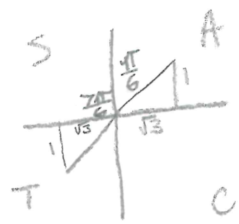
Part C: Thinking and Inquiry

1. Solve in the interval of [0, 2π] : $\frac{\tan(10x) - \tan(4x)}{1 + \tan(10x)\tan(4x)} = \frac{1}{\sqrt{3}}$

$\tan(10x-4x)=\frac{1}{\sqrt{3}}$

$\tan(6x)=\frac{1}{\sqrt{3}}$



$\frac{\pi}{6}=6x$

$\frac{\frac{\pi}{6}}{\frac{6}{1}}=x$

$x=\frac{\pi}{6}\cdot\frac{1}{6}$

$x=\frac{\pi}{12}$

$\frac{7\pi}{6}=6x$

$\frac{\frac{7\pi}{6}}{\frac{6}{1}}=x$

$\frac{7\pi}{6}\cdot\frac{1}{6}=x$

$x=\frac{7\pi}{12}$

∴ $x=\frac{\pi}{12}$ and $x=\frac{7\pi}{12}$.

2. Solve in the interval [π, 2π]: $\cos^2(\frac{x}{16}) - \sin^2(\frac{x}{16}) = \frac{1}{\sqrt{2}}$

$\cos(2(\frac{x}{16}))=\frac{1}{\sqrt{2}}$

$2(\frac{x}{16})=\frac{\pi}{4}$

$\frac{x}{16}=\frac{\pi}{4}/2$

$\frac{x}{16}=\frac{\pi}{8}$

$x=\frac{\pi}{8}\cdot 16$

$x=\frac{16\pi}{8}$

$x=2\pi$

$2(\frac{x}{16})=\frac{7\pi}{4}$

$\frac{x}{16}=\frac{7\pi}{4}/2$

$\frac{x}{16}=\frac{7\pi}{8}$

$x=\frac{7\pi}{8}\cdot 16$

$x=\frac{112\pi}{8}$

∴ In the interval of $[\pi, 2\pi]$, $x=2\pi$.

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3. Develop a formula for $\cos\frac{x}{6}$

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin\left(2\cdot\frac{x}{6}\right) = 2\sin\left(\frac{x}{6}\right)\cos\left(\frac{x}{6}\right)$$

$$\frac{\sin\frac{x}{3}}{2\sin x/6} = \frac{\cancel{2\sin\left(\frac{x}{6}\right)}\cos\left(\frac{x}{6}\right)}{\cancel{2\sin\left(\frac{x}{6}\right)}}$$

$$\cos\frac{x}{6} = \frac{\sin\frac{x}{3}}{2\sin\frac{x}{6}}.$$

$\therefore$ A formula for $\cos\frac{x}{6}$ is

$$\cos\frac{x}{6} = \frac{\sin\frac{x}{3}}{2\sin\frac{x}{6}}.$$

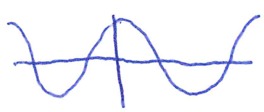
Part D: Communication:

1. Explain how to determine whether a function is even, odd, or neither. You may use words, equations and graphs to aid in your answer.

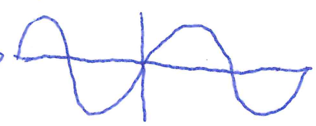
If a function can be reflected on the y-axis and it is symmetrical, the function is even.

If a function can be symmetrical after being reflected on the x and y-axis. it is odd.

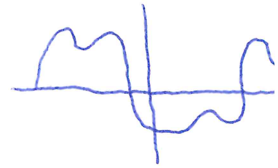
If it isn't symmetrical after either of those, the function is neither even nor odd.



← even. odd →



neither ↓



2. Explain why $\cos^2\theta \neq \cos(2\theta)$. Use an example to justify your response

$\cos^2\theta = \cos(\theta)^2$ whereas $\cos(2\theta) = \cos(\theta+\theta)$

$= \cos\theta\cos\theta$ $\qquad$ $= \cos\theta + \cos\theta.$

Adding is very different from multiplication

$\theta = \frac{\pi}{4}$

LS

$1 - \sin^2\theta = \cos^2\theta$

$1 - \sin^2\frac{\pi}{4} = \cos^2\theta$

$0.4215 = \cos^2\frac{\pi}{4}.$

RS

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos\left(2\cdot\frac{\pi}{4}\right) = \cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{4}$

$\cos\left(2\cdot\frac{\pi}{4}\right) = 0.2372$

$\therefore \cos^2\theta \neq \cos(2\theta).$

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3. Write three different equations that, when graphed, will express identical graphs. Use all a, k, c, and d values at least once.

All answers will vary. This is an example.

$f(x)=3\left(\frac{1}{2}\left(\sin x-\frac{\pi}{3}\right)\right)+2$

$f(x)=3\left(\frac{1}{2}\left(\sin x-\frac{7\pi}{3}\right)\right)+2$

$f(x)=3\left(\frac{1}{2}\left(\sin x-\frac{13\pi}{3}\right)\right)+2$

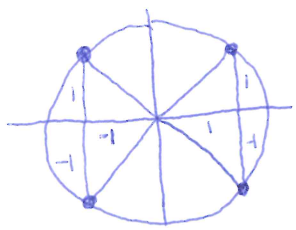
$f(x)=3\left(\frac{1}{2}\left(\sin x-\frac{19}{3}\right)\right)+2$

* Just add period to phase shift or subtract.

4. Explain, with an aid of a diagram, how many solution there are to $\tan^2 x = 1$ and why there are that many.

$\sqrt{\tan^2 x}=\sqrt{1}$

$\tan x=\pm 1$



$\tan^2 x=1$ will have 4 potential solutions. Because of the rule of square roots, when you square root 1, it can be either positive or negative. Causing there to be four potential solutions.

You finally finish the practice test and just in time for your family to arrive!! Lucy got into some of the Christmas cookies while you were not paying attention, oops! But other than that everything is fine. Being very proud of yourself for not allowing your cousins to burn the place down, you say goodbye to everyone, and head home with your parents. You feel accomplished for having been able to complete your mission. You know that you will not turn down babysitting your cousins next time your aunt asks again, knowing that it gave your time to do the practice test you would have procrastinated doing later. Good thing you did it now, right?