Giulia and Josh

Part A: Knowledge & Understanding

1. True & False

a) The degree of the function is the highest exponent in the expression. T

b) A polynomial with degree 6 and a negative leading coefficient will have a graph extending from quadrant ~~2~~ 3 to quadrant ~~1~~ 4. F

c) NOT All polynomial functions are factorable. F

d) For the function $y = -5\,[2\,(x + 3)]^3 + 1$ there is a horizontal translation 3 units to the ~~right~~ LEFT. F

e) For the function $y = -0.5\,[4\,(x - 1)]^3 + 6$ there is a reflection in the x-axis. T

f) For the function $3x^4 - x^2 + 7$ the leading degree is ~~3~~ 4, therefore it is ~~odd~~ EVEN. F

2. Describe the transformations that were applied to $y = x^3$ to create the following function: $y = -4\,[0.5\,(x + 6)]^3 - 2$. State domain and range as well.

REFLECTION IN X AXIS
VERTICALLY STRECHED BY A FACTOR OF 4
HORIZONTALLY TRANSLATED 6 UNITS TO THE LEFT
VERTICALLY TRANSLATED 2 UNITS DOWN
HORIZONTALLY STRECHED BY A FACTOR OF 2

3. Describe the end behaviours of both polynomial functions using the degree and the leading coefficient.

a) $f(x) = x^3 + 2x^2 - 5x + 3$

$x \to -\infty$
$y \to -\infty$

$x \to +\infty$
$y \to +\infty$

b) $f(x) = -4x^6 + 10x^3$

$x \to -\infty$
$y \to -\infty$

$x \to +\infty$
$y \to -\infty$

4. State the remainder when x + 4 is divided into each polynomial.

a) $x^2 + 10x - 18$

$f(-4) = (-4)^2 + 10(-4) - 18$

$= 16 - 40 - 18$

$= -42$

$\therefore$ the remainder is -42

b) $6x^4 + 3x^2 - 9x + 15$

$f(-4) = 6(-4)^4 + 3(-4)^2 - 9(-4) + 15$

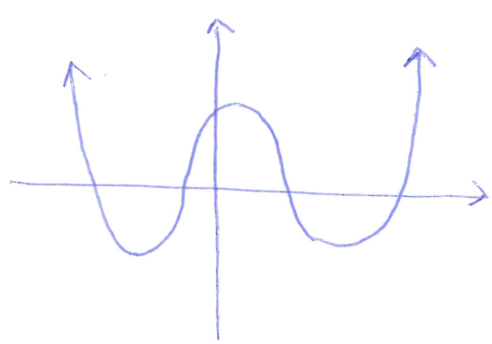
$= 1536 + 48 + 36 + 15$

$= 1635$

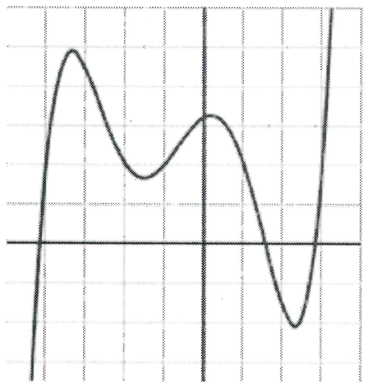
$\therefore$ the remainder is 1635

5. Sketch the graph or state the characteristics of the polynomial function that satisfies each set of conditions.

a) Degree 4, positive leading coefficient, 4 zeros, 3 turning points



b)



DEGREE : ODD

LEADING COEFFICIENT : POSITIVE

ZEROS : 3

TURNING POINTS : 4

6. Divide the polynomial $11x^4 + 7x - 3$ by

a) $2x + 3$

$11x^3 - 33x^2 + 99x - 290$

$x + 3 \overline{)\, 11x^4 + 0x^3 + 0x^2 + 7x - 3}$

$11x^4 + 33x^3$

$0x^4 - 33x^3 + 0x^2 + 7x - 3$

$-33x^3 - 99x^2$

$0x^3 \; 99x^2 + 7x - 3$

$99x^2 + 297x$

$0x^2 - 290x - 3$

$-290x - 870$

$0x + 867$

$11x^4 + 7x - 3 = (x+3)(11x^3 - 33x^2 + 99x - 290) + 867$

b) $x^2 - 1$

$11x^2 + 11$

$x^2 + 0x - 1 \overline{)\, 11x^4 + 0x^3 + 0x^2 + 7x - 3}$

$11x^4 + 0x^3 - 11x^2$

$0 \quad 0 \quad + 11x^2 + 7x - 3$

$11x^2 + 0x - 11$

$7x + 8$

$11x^4 + 7x - 3 = (x^2 - 1)(11x^2 + 11) + 7x + 8$

Part B: Application

1. Determine the value of k such that P(x) = kx^3+4x^2-3x-4 have the same remainder when it is divided by x + 1 and x - 2 .

$P(-1) = P(2)$

$P(-1) = -k + 4 + 3 - 4 = 3 - k$

$P(2) = 8k + 16 - 6 - 4 = 6 + 8k$

$3 - k = 6 + 8k$

$9k = -3$

$k = -\frac{1}{3}$

2. When $x^4 + ax^3 + bx^2 + x - 1$ is divided by x - 1, the remainder is 3. When it is divided by ~~x^2~~ - x +2, the remainder is -3. Find *a* and *b*.

$P(1) = 3$

$P(1) = 1 + a + b + 1 - 1$

$a + b + 1 = 3$

$a + b = 2$

$P(2) = -3$

$P(2) = 16 + 8a + 4b + 2 - 1$

$8a + 4b + 17 = -3$

$\frac{8a}{4} + \frac{4b}{4} = \frac{-20}{4}$

$2a + b = -5$

$b = -5 - 2a$

$a + b = 2$

$b = -5 - 2a$

$a - 5 - 2a = 2$

$-a = 7$

$a = -7$

$a + b = 2$

$-7 + b = 2$

$b = 9$

$a = -7$

$b = 9$

3. Full factor the polynomial $8x^3+12x^2+6x+1$

$8x^3+1 + 12x^2+6x$

SUMS OF CUBES

$(2x+1)(4x^2-2x+1) + 6x(2x+1)$

$(2x+1)(4x^2-2x+1+6x)$

$(2x+1)(4x^2+4x+1)$

$(2x+1)(2x+1)^2 = (2x+1)^3$

4. The population of a certain city in Northern Ontario can be modelled by

$$P(t)=65-0.0025t^3+3.32t$$

where the population, P, is in thousands and the time, t, is in years from 2014.

a. What will be the population of the city in 2020?

~~b. In what year will the population reach 75000?~~

c. Based on the equation described what will happen to the population over time

a) $2020-2014=6$

$t=6 \quad P(6)=65-0.0025(6)^3+3.32(6) = 84.38$

$\downarrow$

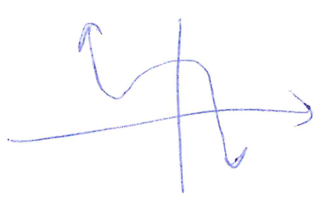
84380 people

b) ~~$P=75000 \rightarrow 75$~~

~~$75 = 65 - 0.0025t^3 + 3.32t$~~

~~$10 = 3.32t - 0.0025t^3$~~

c) DECREASE OVER TIME



$\rightarrow$ CUBIC FUNCTION WITH A NEGATIVE LEADING COEFFICIENT

5. The volume of a crate containing organic produce is ($x^4 + 5x^3 - 7x^2 - 5x + 6$)$cm^3$. The crate is ($x+6$) cm long and ($x+1$) cm wide. How high is the crate?

$V = l \cdot w \cdot h$

$(x+6)(x+1)h = x^4 + 5x^3 - 7x^2 - 5x + 6$

$(x^2 + 7x + 6)h = x^4 + 5x^3 - 7x^2 - 5x + 6$

$$\begin{array}{r|l} & x^2 - 2x + 1 \\ x^2 + 7x + 6 & x^4 + 5x^3 - 7x^2 - 5x + 6 \\ & x^4 + 7x^3 + 6x^2 \\ & 0 \quad -2x^3 - 13x^2 - 5x + 6 \\ & -2x^3 - 14x^2 - 12x \\ & 0 \quad x^2 + 7x + 6 \\ & x^2 + 7x + 6 \\ & 0 \end{array}$$

$h = x^2 - 2x + 1$

6. Factor the difference and sum of cubes functions below:

a) $216x^3 + 1$

$(6x+1)(36x - 6x + 1)$

b) $x^3y^6 - 64$

$(xy^2 - 4)(x^2y^4 + 4xy^2 + 16)$

Part C: Thinking/Inquiry/Problem Solving

1. For what value will the polynomial $f(x) = 3x^3 + kx - 15$ have the opposite remainder when it is divided by x-2 and x+3.

$f(2) = -f(-3)$

$f(2) = 3(8) + 2k - 15 = 24 - 15 + 2k = 2k + 9$

$f(-3) = 3(-27) - 3k - 15 = -96 - 3k$

$2k - 9 = 96 + 3k$

$k = -105$

2. When $x^3 + mx^2 - nx + 16$ is divided by x -2 , the remainder is 40 and x - 1 is a factor. Determine the values of m and n.

$f(2) = 40$

$8 + 4m - 2n + 16 = 40$

$4m - 2n = 16$

$2m - n = 8$

$n = 2m - 8$

$f(1) = 0$

$1 + m - n + 16 = 0$

$m - n = -17$

$n = m + 17$

$m = 25$

$n = 42$

$2m - 8 = m + 17$

$m = 25 \rightarrow n = m + 17 = 25 + 17 = 42$

3. Determined the dividend when the divisor is x + 3, the quotient is $x^4 + 2x^3 - x + 7$ and the remainder is 8.

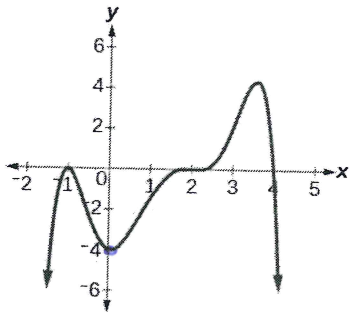
DIVIDEND = DIVISOR · QUOTIENT + REMINDER

$(x+3)(x^4 + 2x^3 - x + 7) + 8$

$x^5 + 2x^4 - x^2 + 7x + 3x^4 + 6x^3 - 3x + 21 + 8$

$x^5 + 5x^4 + 6x^3 - x^2 - 4x + 29$

4. Determine the equation of the graph f(x)



$y = a(x+1)^2(x-2)^3(x-4)$

y intercept $= -4$

$-4 = a(0+1)^2(0-2)^3(0-4)$

$-4 = 32a$

$a = -\frac{1}{8}$

$y = -\frac{1}{8}(x+1)^2(x-2)^3(x-4)$

Part D: Communication

1. Use an example to show how synthetic division and regular polynomial division are essentially the same. Show what the difference is as well.

SAME

$$\begin{array}{c|cccc} 3 & 3 & -5 & -7 & -1 \\ & & 9 & 12 & 15 \\ \hline & 3 & 4 & 5 & 14 \end{array}$$

Coefficients of quotient

$3x^2+4x+5$

$x-3 \overline{)\,3x^2-5x^2-7x-1}$

$3x^2-9x^2$

$4x^2-7x$

$4x^2-12x$

$5x-1$

$5x-15$

14

~~[illegible] not possible to really [illegible]~~

REMAINDERS

DIFFERENCE

$(x^2)+3$

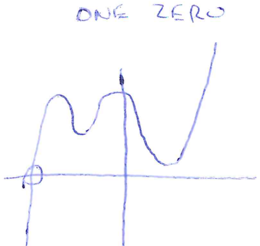
↳ This will not work in synthetic division

2. Write an equation for f(x) = x^{12} if it carries a reflection in the y-axis, is stretched vertically by a factor of 2, and is translated 5 units up and 2 units to the left.

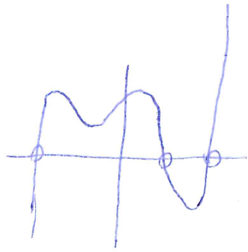
$f(x) = 2[-1(x+2)]^{12} + 5$

3. Show how a quintic function can have one, three, and five zeros using diagrams.

ONE ZERO



THREE ZEROS



FIVE ZEROS