McKenna

Practice Test: Rational Functions and Equations

Fill In The Blanks:

1. At each zero of the original function, the reciprocal function has a(n) asymptote.
2. Intervals of increase on the original function are intervals of decrease on the reciprocal function. Intervals of decrease on the original function are intervals of increase on the reciprocal function.
3. The function is undefined where the denominator is equal to Ø.
4. The equation of the vertical asymptote comes directly from the zero of the denominator.
5. A rational function has a hole when p(x) and q(x) contain a common factor.
6. An oblique asymptote occurs when the degree is greater on top by exactly 1.

Part A: Knowledge

1. Given the function $f(x) = 4 - x^2$

a) Determine the domain and range, intercepts, positive/negative intervals, and increasing/decreasing intervals.

b) Use your answers for part a) to sketch the graph of the reciprocal function.

D: $\{x \in \mathbb{R}\}$ R: $\{f(x) \in \mathbb{R} \mid f(x) \leq 4\}$

x-int: $x = 2$ and $x = -2$

y-int: $y = 4$

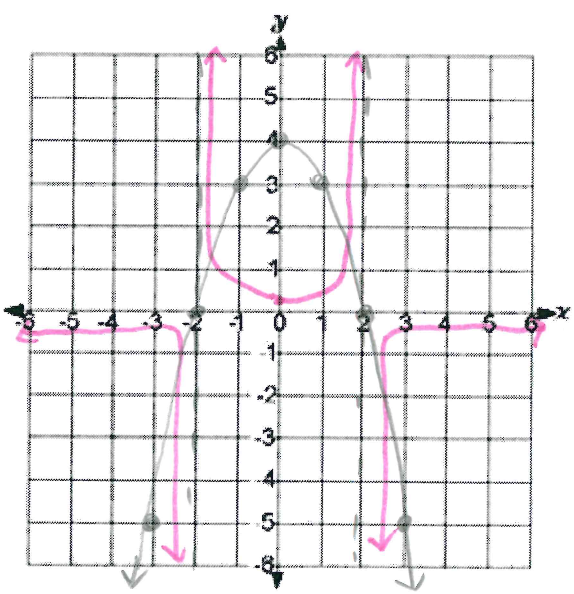
$\oplus (-2, 2)$

$\ominus (-\infty, -2)$

$\ominus (2, +\infty)$

$\uparrow (-\infty, 0)$

$\downarrow (0, +\infty)$



2. Solve the equations below.

a) $\frac{x+3}{x-2} = \frac{x+4}{x-5}$

$x+3(x-5) = x+4(x-2)$

$\cancel{x^2} - 5x + 3x - \cancel{15} = \cancel{x^2} - 2x + 4x - 8$

$-x^2 + 5x - 3x + 15$

$0 = 4x + \cancel{7}$

$\frac{-7}{4} = \frac{4x}{4}$

$x = \frac{-7}{4}$

b) $\frac{x-8}{x+10} = \frac{x-4}{x+6}$

$x-8(x+6) = x-4(x+10)$

$\cancel{x^2} + \cancel{6x} - \cancel{8x} - \cancel{48} = \cancel{x^2} + 10x - 4x - 40$

$-x^2 - 6x + 8x + 48$

$0 = \frac{8x}{8} + \frac{8}{8}$

$0 = 8(x+1)$

$\therefore$ when $x = 8$ and $x = -1$

3. For the function, $f(x) = \frac{1}{x^2-2x-15}$

a) List the x- and y- intercepts and determine the equations of any asymptotes.

vertical asymptote @ $x = -3$ and $x = 5$

horizontal asymptote @ $y = 0$

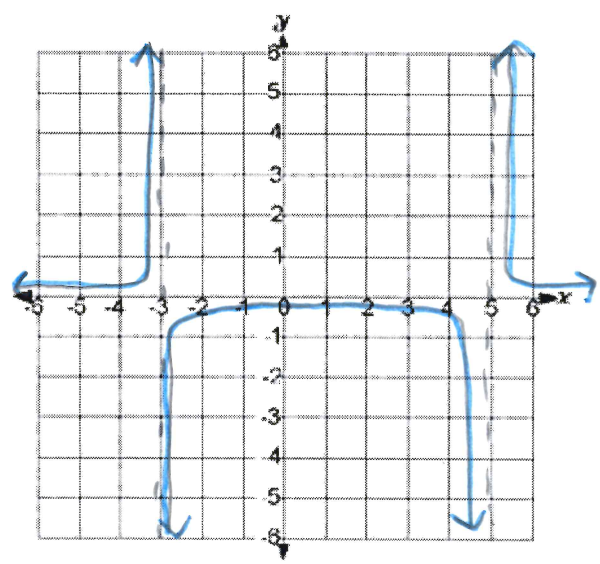
x-int: no x-intercepts

y-int: $\frac{-1}{15}$

b) List the intervals over which the function is decreasing

$\downarrow (1, 5)$

$\downarrow (5, +\infty)$



c) Sketch the graph using the information you've found so far on the axes above.

4. For each function, determine the equations of any vertical asymptotes, any horizontal asymptotes (other than the x-axis), the locations of any holes, and the existence of any oblique asymptotes.

a) $f(x) = \frac{x}{x-4}$

vertical asymptote: $x = 4$
horizontal asymptote: $y = 1$

c) $h(x) = \frac{2x+7}{2x+4}$

$\frac{2}{2} = 1$

horizontal asymptote: $y = 1$
vertical asymptote: $0 = 2x + 4$
$x = -2$

$\frac{-4}{2} = x$
$-2 = x$

b) $g(x) = \frac{x^2-16}{x+4}$

$= \frac{\cancel{(x+4)}(x-4)}{\cancel{x+4}}$

$0 = x + 4$
$-4 = x$
∴ hole @ $x = -4$

d) $j(x) = \frac{x^2+5}{x-3}$

$$\begin{array}{r} x + 3 \\ x - 3 \overline{) x^2 + 0x + 5} \\ x^2 - 3x \\ \hline 3x + 5 \\ 3x - 9 \\ \hline 14 \end{array}$$

∴ oblique asymptote @ $x + 3$.

Part B: Application

1. After a shipwreck in the waters of New Brunswick, oil has been leaking into the ocean at a constant rate. It is vital that the oil is cleaned up before 80 minutes has passed in order to keep the marine life healthy. The spill can be modelled by the function $V(t) = \frac{32\,000t}{24+t}$ (V, in litres, t, in minutes). At this point 24320L of oil has been spilt. Determine whether or not they were able to save the marine life in time (is the time less than 80 minutes).

$v(t) = \frac{32\,000t}{24+t}$

$24320 = \frac{32000t}{24+t}$

$24320(24+t) = 32000t$

$583680 + \cancel{24320t} = 32000t$
$- 24320t$

$\frac{583680}{7680} = \frac{768\cancel{0}t}{\cancel{7680}}$

$76 = t$

$76 < 80$ ∴ yes they were able to save the marine life.

2. Use a full algebraic solution to find the solution:

$$\frac{x}{x-3} \leq \frac{-8}{x-6}$$

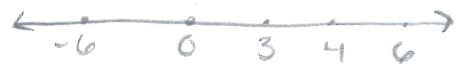
$$0 \leq \frac{-8(x-3)}{(x-6)(x-3)} - \frac{x(x-6)}{(x-3)(x-6)}$$

$$0 \leq \frac{-8x+24}{(x-6)(x-3)} - \frac{x^2-6x}{(x-6)(x-3)}$$

$$0 \leq \frac{-x^2-2x+24}{(x-6)(x-3)}$$

$$0 \leq \frac{-(x^2+2x-24)}{(x-6)(x-3)}$$

$$0 \leq -\frac{(x-4)(x+6)}{(x-6)(x-3)}$$



	$(-\infty,-6)$	$(-6,3)$	$(3,4)$	$(4,6)$	$(6,+\infty)$
$(x-4)$	−	−	−	+	+
$(x+6)$	−	+	+	+	+
$(x-6)$	−	−	−	−	+
$(x-3)$	−	−	+	+	+
	−	−	+	+	+

$\therefore \frac{x}{x-3} \leq \frac{-8}{x-6}$ when $(3,4)$, $(4,6)$ and $(6,+\infty)$

3. Determine the equation of the oblique asymptote of the function below.

$$\frac{-3x^2+2}{x-1}$$

$$\begin{array}{r|l} & -3x-3 \\ \hline x-1 & -3x^2+0x+2 \\ & -3x^2+3x \\ \hline & \cancel{0x^2}-3x+2 \\ & -3x+3 \\ \hline & -1 \end{array}$$

$\therefore$ the eq'n of the oblique asymptote is $-3x-3$

4. Find the domain, intercepts, asymptotes, asymptote approach and positive/negative intervals. Sketch the graph.

$f(x) = \frac{x+6}{2x+4}$

horizontal asymp: $y = \frac{1}{2}$

vertical asymp: $x = -2$

y-int: $\frac{6}{4} = 1.5$

x-int: $x = -6$

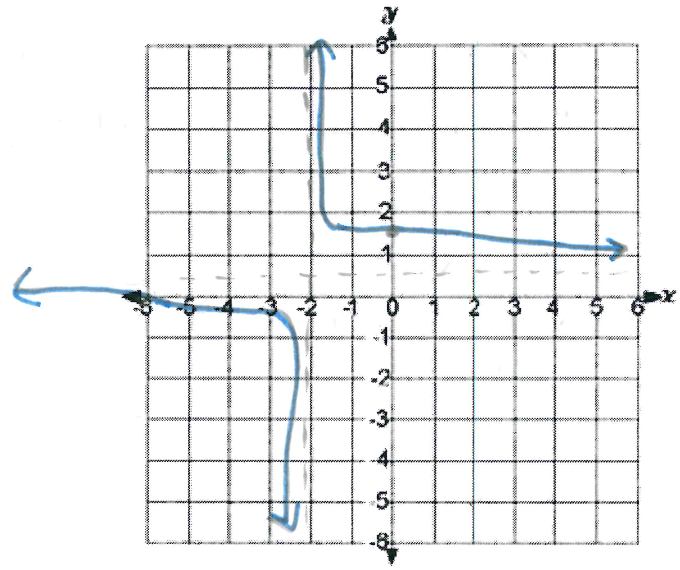
as $x \to -2$ from left $y \to -\infty$

as $x \to -2$ from right $y \to +\infty$

$\oplus\ (-\infty, -6)$

$\oplus\ (-2, +\infty)$

$\ominus\ (-6, -2)$



5. An aquarium is receiving a new sea mammal and have begun preparing the tank. To do this they must turn the water into a saltwater environment for the mammal to adjust properly. The concentration of salt, c, in the tank at t minutes is given by $c(t) = \frac{20t}{300+t}$ where c is measured in mg/L. After how many minutes does the tank reach a salt concentration of 15 mg/L?

$15 = \frac{20t}{300+t}$

$15(300+t) = 20t$

$4500 + 15t = 20t$
$\quad -15t \quad -15t$

$\frac{4500}{5} = \frac{5t}{5}$

$900 = t$

$\therefore$ after 900 minutes (15 hours) the tank reaches a concentration of 15 mg/L

Part C: Thinking/Inquiry

1. Sally received a large order of t-shirts for \$360. She gave 15 shirts to her best friend and then sold the rest at her school for a fundraiser. Her total earnings was \$300, making a profit of \$12 on each shirt. How many shirts originally came in the box? What was the original price of each shirt?

let x represent # of t-shirts

\$ paid: $\frac{360}{x}$ \$ made: $\frac{300}{x-15}$ profit: \$12 each

$$\frac{360}{\cancel{x}} + \frac{300}{\cancel{x-15}} = 12$$

$$360(x-15) + 300(x) = 12(x)(x-15)$$

$$360x - 5400 + 300x = 12x^2 - 180x$$

$$-660x + 5400$$

$$0 = \frac{12x^2}{12} - \frac{840x}{12} + \frac{5400}{12}$$

$$0 = 12(x^2 - 70x + 450)$$

$$\frac{70 \pm \sqrt{(-70)^2 - 4(1)(450)}}{2}$$

$+$: $= 62.83$ too high

$-$: $= 7.16$

∴ the original price for each shirt is about 7 dollars and there was 51 shirts in the box.

2. A function of the form $f(x) = \frac{ax+b}{cx+d}$ has the following features:

- x-intercept at 4
- y-intercept at 2
- vertical asymptote x = -2
- horizontal asymptote y = -1

Determine an equation for this function. Fully explain how you came to your answer.

to determine the eq'n I worked backwards. I knew that when the numerator was set to zero $x = 4$ so $0 = -3x + 12$

$$\frac{-12}{-3} = x$$

$$4 = x$$

y-int must be 2 so I figured out which ratio with −12 was the same $\frac{-12}{-6} = 2$. When the denominator $= \varnothing$ $x = -2$ so $0 = 3x + 6$

$$\frac{-6}{3} = x$$

$$-2 = x$$

and the leading coefficients must divide to −1 so $\frac{-3}{3} = -1$

$$y = \frac{-3x + 12}{3x + 6}$$

3. Ms. Humphries has been documenting the tree frog and bullfrog population in her backyard for the past 4 years. Because she is so mathy she came up with an equation to represent both populations. The tree frog population can be represented by $f(t) = t - 3$, where $f(t)$ is the total tree frog population and t is the number of months. The bullfrog population can be modelled by $b(t) = \frac{18}{t}$. Determine the time period when the population of tree frogs is LESS than the population of bullfrogs.

$t - 3 < \frac{18}{\cancel{t}}$

$t - 3(t) < 18$

$t^2 - 3t - 18 < 0$ $x: -18$, $+: -3$; -6, 3

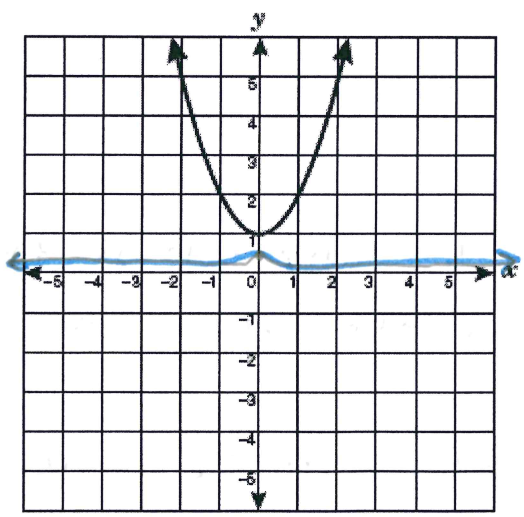
$(t+3)(t-6) < 0$

$t = -3$ $t = 6$

∴ the pop. of tree frogs is less than the pop. of bull frogs for the first 6 months (Jan-June)

4. Write a possible equation for the reciprocal of this function and then sketch it on the graph.

$Y = \frac{1}{x^2+1}$



5. When Jack and McKenna work together in the pharmacy they can fill 40 prescriptions in 46 minutes. When McKenna works alone she can finish the prescriptions in 11 minutes less time than Jack can when he works alone. When Jack works alone, how long does he take to fill 40 prescriptions? Round your answer to the nearest whole number.

Total: 46 McK: $(j-11)$ Jack: j

consider fraction fills per min

$$\frac{1(46)(j-11)}{\cancel{j}} + \frac{1(j)(46)}{j-\cancel{11}} = \frac{1(j)(j-11)}{\cancel{46}}$$

$$46j - 506 + 46j = j^2 - 11j$$

$$92j - 506 = j^2 - 11j$$

$$0 = j^2 - 103j + 506$$

$$j = \frac{103 \pm \sqrt{(-103)^2 - 4(1)(506)}}{2}$$

$+$ ↙ ↘ $-$

$= 97.8$ $= \cancel{5.17}$

too low

∴ Jack takes about 98 mins (1.6 hours) to fill 40 prescriptions.

Part D: Communication

1. Write a detailed explanation of the steps required to graph a reciprocal function using the graph of the original function. Explain what each step gives you and provide an example.

1) Find the asymptotes of the function by determing the zeros

2) The Intervals where the function is +ve or −ve will be the same on the reciprocal.

3) The reciprocal function will be increasing where the original function is decreasing and vice versa.

2. Explain when a rational function will have an oblique asymptote. Use words, pictures and/or examples from this unit to fully explain your answer.

$\frac{P(x)}{q(x)}$

A rational function has holes where p(x) and q(x) have a common factor

ex. $\frac{x^2-9}{x+3}$

$= \frac{\cancel{(x+3)}(x-3)}{\cancel{x+3}}$ $\therefore$ hole @ $x = -3$

$0 = x+3$

$-3 = x$

3. Why do the graphs of the reciprocal of linear functions always have vertical asymptotes but the reciprocals of quadratic functions sometimes do not? Provide sketches of 3 different reciprocal functions to explain your answer.

A reciprocal function has a vertical asymptote when the denominater is equal to zero. A quadratic function in the form of $x^2 + b$ has no real zeros therefore will not have a vertical asymptote.